Reduced-Rank Array Modes of the California Current Ocean Observing System

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Outline

• Array modes – an idea borrowed from antenna theory
• Reduced-rank Array Modes (RAMs)
• RAMs of the California Current observing system
• Overfitting of DA to the data – a useful stopping criteria
• How effective are the observations in constraining the background errors?
Characteristic Modes of an Antenna

Resonance $\propto \frac{1}{|\lambda - \omega|}$

Oceanographic applications:
- Bennett (1985)
- Le Hénaff et al (2009)

Response $\propto \frac{1}{\lambda}$

Current distribution and radiation pattern for a characteristic mode with a resonant frequency of 4.1GHz
Array Modes: Assessing the Efficiency of the CA Current Observing System

- How well does the CA Current observing system “observe” the circulation given our prior hypotheses about the errors?

**CCS observing system:**
- Satellite SST – daily (AVHRR, AMSR, MODIS)
- Aviso gridded SSH - daily
- *In situ* T & S profiles

**ROMS CCS domain**

**In situ T & S**

4 Jan – 18 April 2003
The Analysis Equation

• Recall the analysis equation:

\[ x_a = x_b + BG^T \left( GBG^T + R \right)^{-1} (y - H(x_b)) \]

Analysis increment

• Solved iteratively in ROMS 4D-Var system using the Lanczos formulation of the B-preconditioned RPCG.
**Reduced Rank** **Array Modes (RAMs)**

In **ROMS 4D-Var** the analysis equation is solved using the Lanczos algorithm:

\[ x_a = x_b + BG^T V_m T_m^{-1} V_m^T GBG^T R^{-1} (y - H(x_b)) \]

This can be rewritten as:

\[ x_a = x_b + \sum_{i=1}^{m} \alpha_i \Psi_i \quad \text{where} \quad \Psi_i = BG^T V_m u_i \]

\[ (\lambda_i, u_i) \] are the eigenpairs of \( T_m \)

\( \alpha_i = \lambda_i^{-1} u_i^T V_m^T G B G^T R^{-1} (y - H(x_b)) \)

\( \Rightarrow (R^{-1}G B G^T + I)^{-1} m=\# \text{ of inner-loops} \)

**EOFs of** \((R^{-1}G B G^T + I)\)

**NOTE:** The array modes depend **ONLY** on the obs locations, and **NOT** the obs values.
RAM amplitudes of the California Current system

31 year sequence of 4D-Var analyses (1980-2010)
8 day overlapping windows
Obs assimilated: SST, SSH, in situ T and S
1 outer-loop, 14 inner-loops
14 March, 2001

The sub-space activated by the obs & d.f.s

U : 14 March

V : 14 March

RAM \(\Psi_1\)

Temperature : 14 March

Salinity : 14 March
Overfitting of the Model to the Observations

Contribution of each array mode to the SST increment on 14 March 2001: recall $m=14$ inner-loops.

The Bennett and McIntosh (1984) “1% rule”: Discard array modes with eigenvalues 1% or less of the max value.
The “1% Rule” to avoid overfitting to the Observations

Average eigenvalue ratio for all cycles vs number of inner-loops employed:
(suggests we should use no more than 10 inner-loops to prevent overfitting of the observations)
RMS SST difference (14 inner-loops minus 10 inner-loops) for 2001
RAMs: Assessing the Efficiency of the CA Current Observing System

- How effective is the CA Current observing system at constraining the circulation given our prior hypotheses about the errors B and R?

CCS observing system:
- Satellite SST – daily (AVHRR, AMSR, MODIS)
- Aviso gridded SSH - daily
- In situ T & S profiles
The Analysis Equation

• Recall the analysis equation:

\[
X_a = X_b + B G^T \left( G B G^T + R \right)^{-1} \left( y - H(x_b) \right)
\]

Therefore, to reduce errors in \( x_b \), the observing system must effectively observe (directly via \( G \) or indirectly via \( G^T \)) the dominant EOFs of \( B \).
Projection of RAMs on the EOFs of B

$B$ is $O(10^6 \times 10^6)$

Number of EOFs required to recover RAMs $<< \dim(B)$

$\Rightarrow$ the observing system poorly constrains much of the space spanned by $B$
Summary & Conclusions

- The RAMs identify the sub-space informed by the observations during 4D-Var
- RAMs can be computed from archived 4D-Var output
- RAMs with small $\lambda$ will amplify obs errors
- A useful practical stopping criteria identified for 4D-Var
- The current array observes the EOFs of $\textbf{B}$ rather poorly
- Pseudo-observations could be used to optimize the observing system