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# Assimilating sparse historical data using large scale EOF error covariances

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# Outline

- Motivation
- New NEMOVAR
- Diffusion modelled covariance
- Ensemble covariance
- EOF covariance



# Motivation

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ERA Clim 2 coupled reanalysis planned from 1900 to approx present day

Problem the ocean observation network has changed markedly in the last 100 years or so.

Observations were sparse and sampling was inhomogeneous.

Observations now much less sparse and more globally homogeneous, but we still have sparse sampling at depth

How can assimilation make best use of the sparse historical data while still doing a good job with today's data?

The key thing which gives data assimilation its power is the background error covariance which allows us to spread information from the observation locations.

Can we improve the error covariance structures to allow us to correctly spread sparse observation information over greater distances in order to fill in the gaps?



# NEMOVAR what is it

- Ocean data assimilation system for NEMO (3D/4D variational )
- It's the result of a collaboration between CERFACS, ECMWF, INRIA, Met Office ...
- At the Met Office this is used in FOAM (deep and shelf ocean) and OSTIA
- Main benefits: Compatible with the NEMO. It works with the ORCA grids natively. Efficient. There's an effective balance operator.



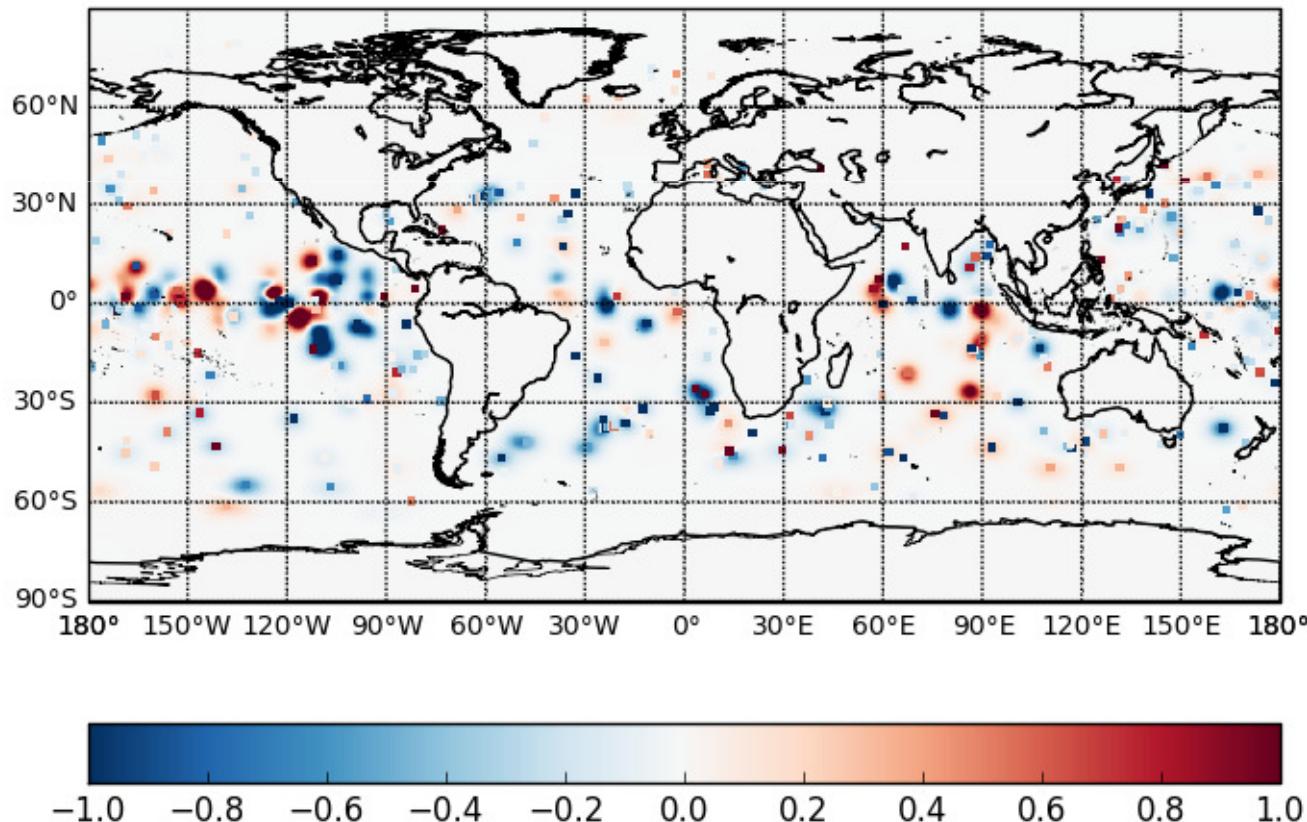
# New hybrid B NEMOVAR – what's new?

- Recently there has been a major improvement to NEMOVAR (mainly developed at CERFACS thanks to Anthony Weaver) (v4)
- We are still using an earlier version (v3) operationally
- The major changes from v3 to v4 are to improve the spatial spreading of observation information:
  - 2D implicit solver for correlation modelled by diffusion
  - Allows the use of ensemble information
  - Also the use of EOFs (by me!)

# Some example Increments

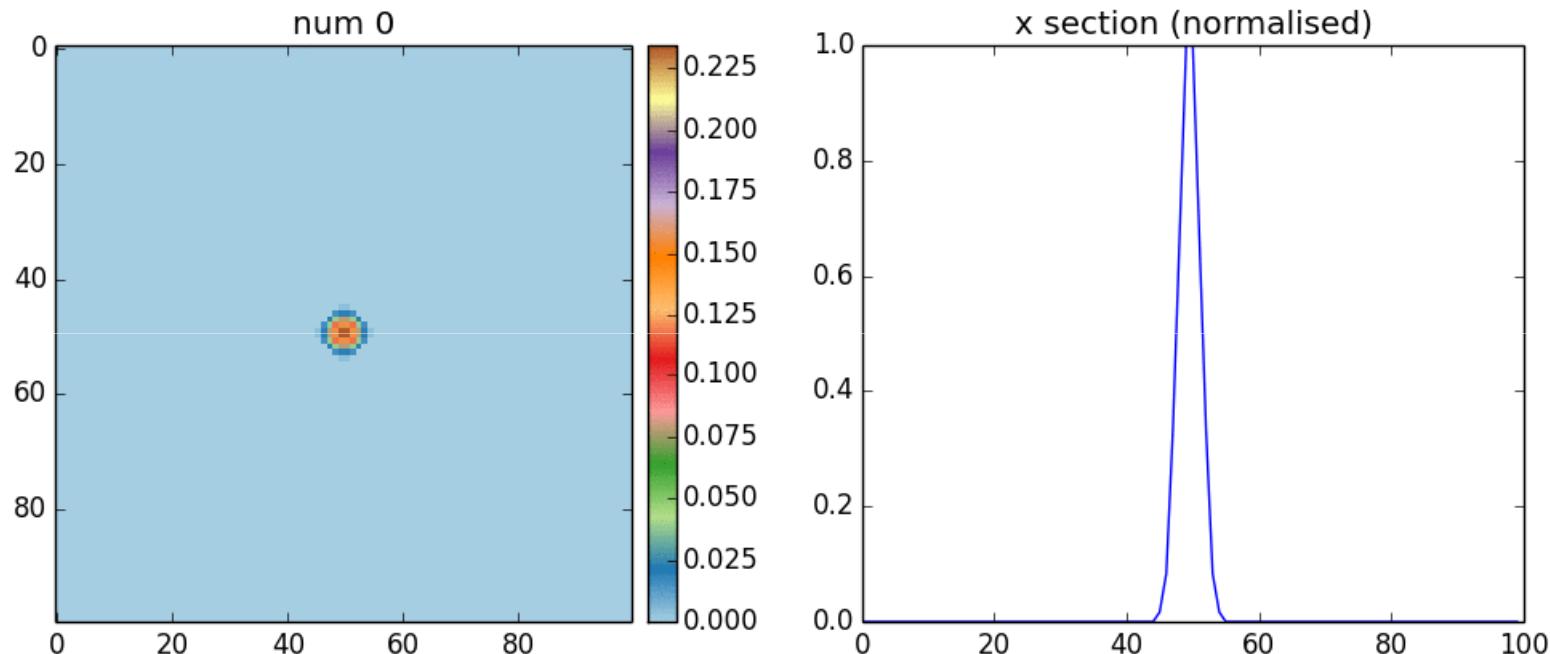
using just the diffusion modelled covariances

POTM:omb none





## 2D diffusion (explicit – Dirichlet BCs)

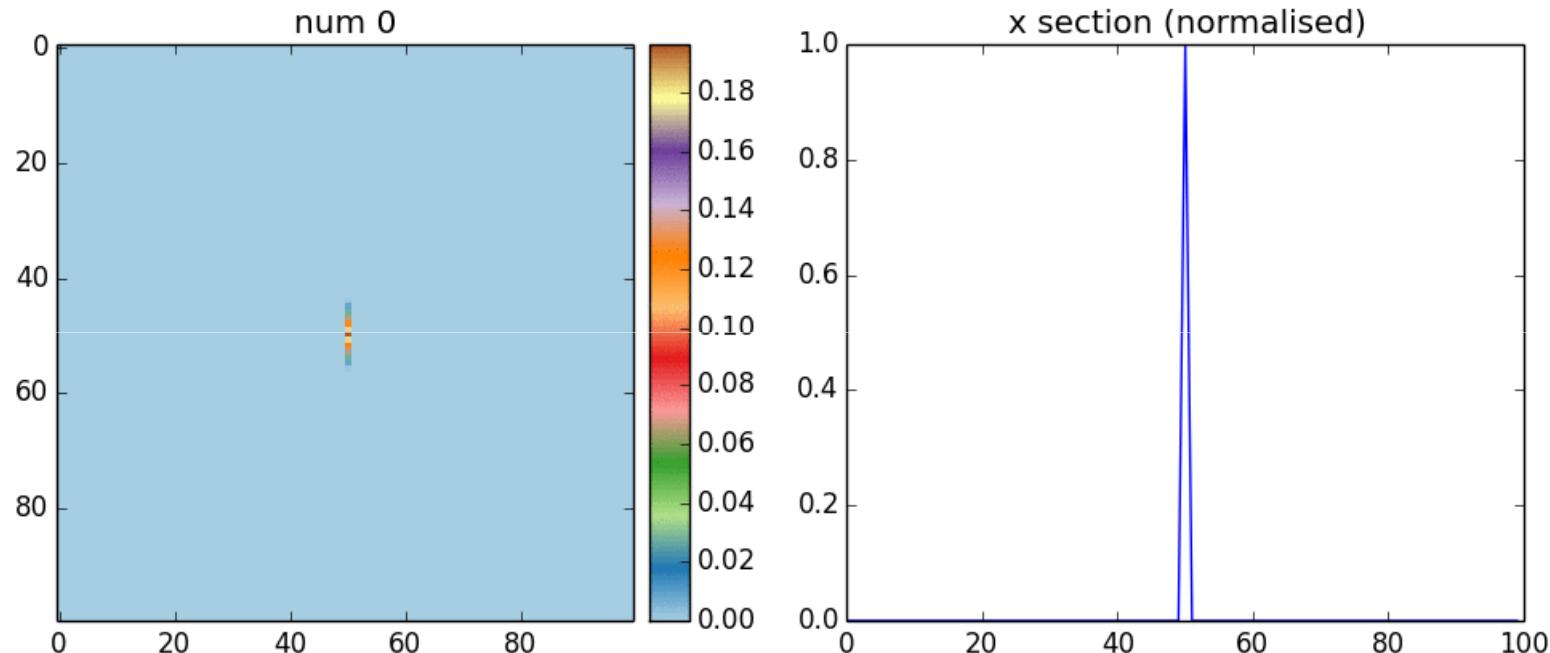


Toy example using python

Implicit solver for 2D diffusion used in New (v4) NEMOVAR



# 2x1D diffusion model (explicit)



Implicit version of this used in v3 (old) NEMOVAR



## Ensemble covariance

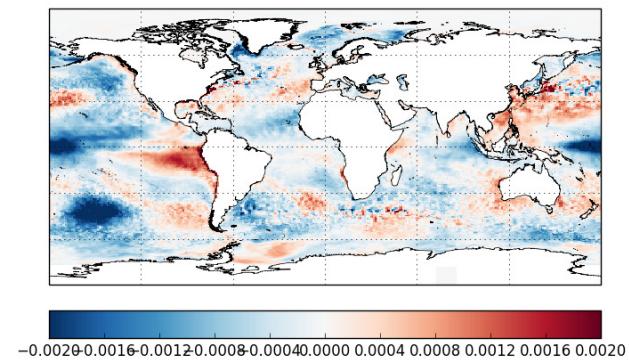
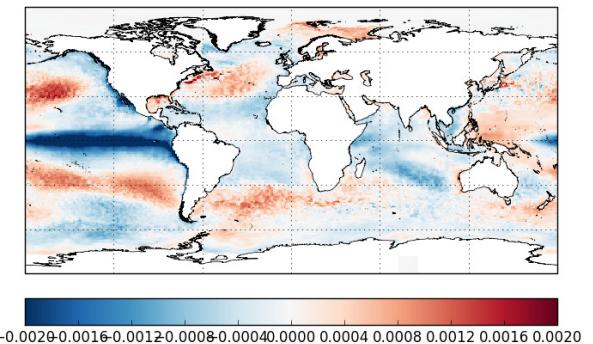
The diffusion model gives a rather limited representation of the real covariance. Another idea is to use information from an ensemble of model runs to spread observation information in a flow dependent way.

This is akin to working in ensemble space rather than real space



# EOF DA

- I've adapted the ensemble covariance code for EOF DA. EOFs are just another “space” to spread the observation information in.
- This work is motivated by the sparseness of historical data. This is for ERA-CLIM2 – ECMWF coupled reanalysis R&D - but may also be applicable to the decadal prediction system.
- Top two SST EOFs from GloSea 5 reanalysis [>>>](#)





# EOF DA

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## Working in model space

$$J(\delta \mathbf{x}) = \frac{1}{2} \delta \mathbf{x}^T \mathbf{B}^{-1} \delta \mathbf{x} + \frac{1}{2} (\mathbf{y} - \mathbf{H}(\mathbf{x}_b + \delta \mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}(\mathbf{x}_b + \delta \mathbf{x}))$$

$$\delta \mathbf{x} = \mathbf{E} \mathbf{a}$$

$\mathbf{a}$ = vector of coefficients/weights for each EOF  
(Temperature anomaly =  $\mathbf{E}\mathbf{a}$ )

$\mathbf{E}$ = EOFs

$\Lambda$ = diagonal matrix of eigenvalues (calculated with the EOFs) (squared)

## Working in EOF space

$$J(\mathbf{a}) = \frac{1}{2} \mathbf{a}^T \Lambda^{-1} \mathbf{a} + \frac{1}{2} (\mathbf{y} - \mathbf{H}(\mathbf{x}_b + \mathbf{E}\mathbf{a}))^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}(\mathbf{x}_b + \mathbf{E}\mathbf{a}))$$

## Hybrid

$$\delta \mathbf{x} = w_1 \mathbf{E} \mathbf{a} + w_2 \delta \mathbf{x}_{\text{residual}}$$

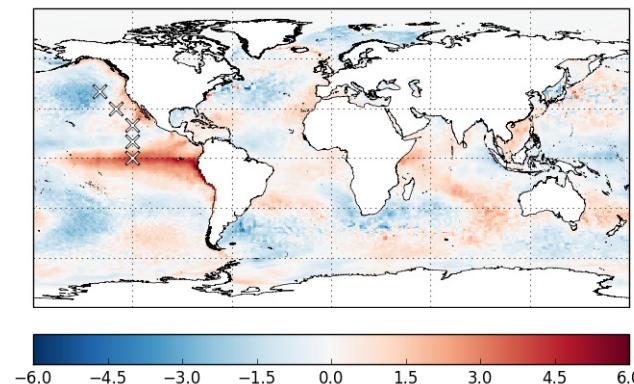
$$J = w_1 \frac{1}{2} \mathbf{a}^T \Lambda^{-1} \mathbf{a} + w_2 \frac{1}{2} \delta \mathbf{x}_{\text{res}}^T \mathbf{B}^{-1} \delta \mathbf{x}_{\text{res}} + [\text{obs cost}]$$



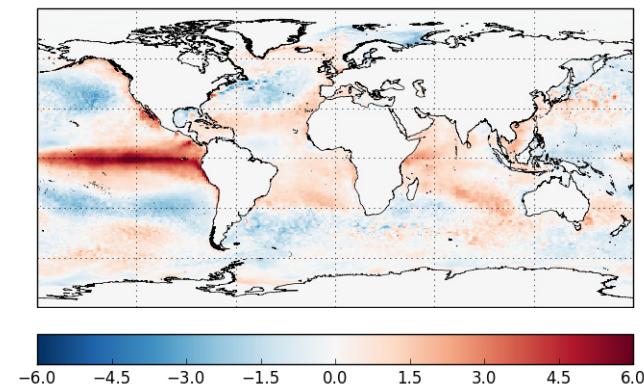
# Increments with different hybrid EOF and diffusion modelled covariance weights

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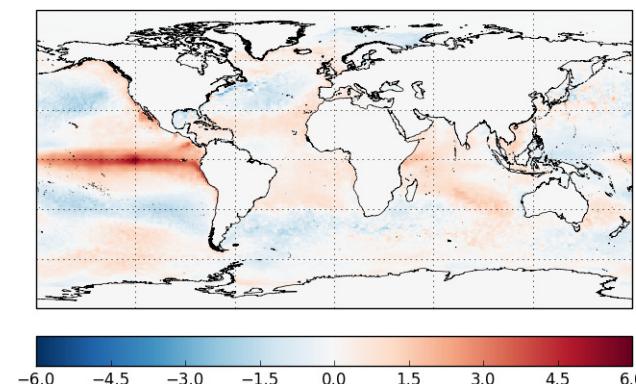
Truth and sim. obs locations



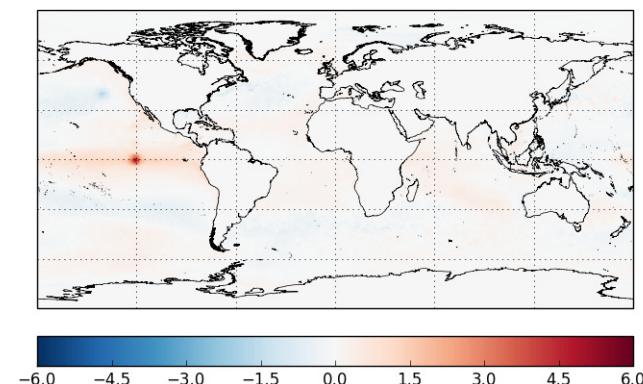
$W_{\_1}=1$       ( $W_{\_1}+W_{\_2}=1$ )



$W_{\_1}=0.01$



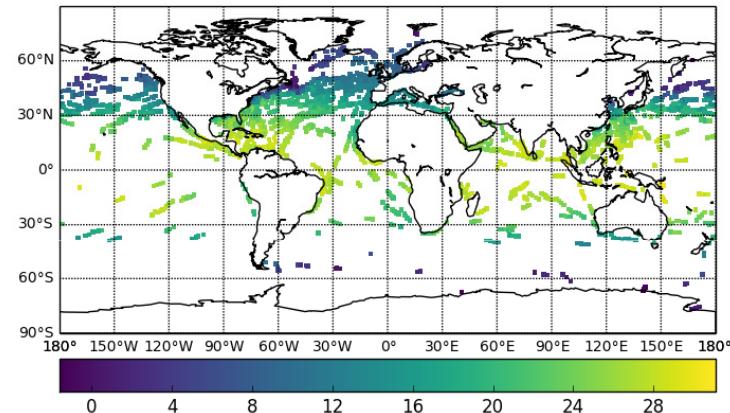
$W_{\_1}=0.001$



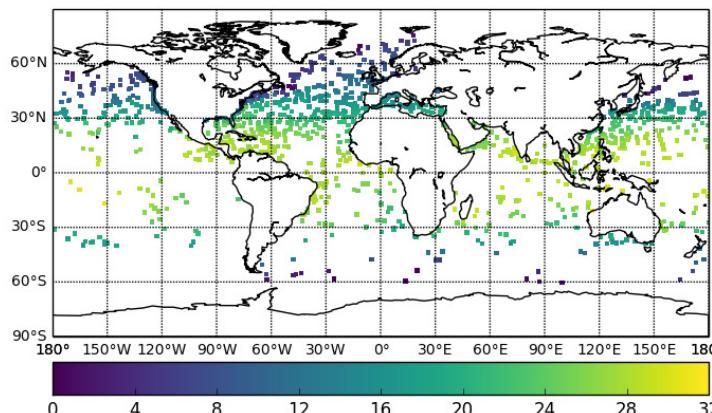
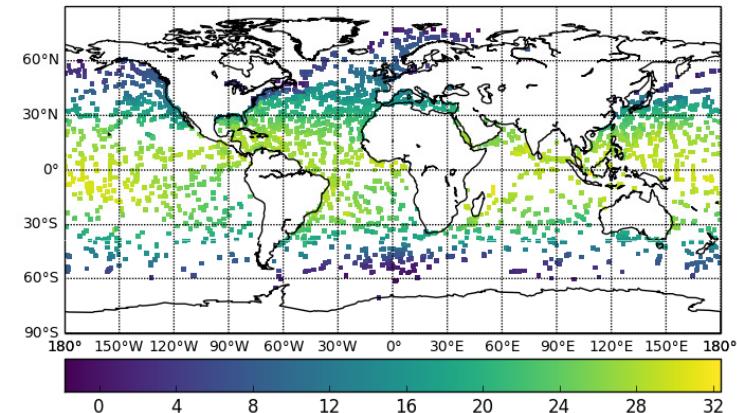
# Observing system experiment

Subsample modern day observations to look like historical data (example of in-situ SST observations in °C)

1 Jan 1960



1 Jan 2010

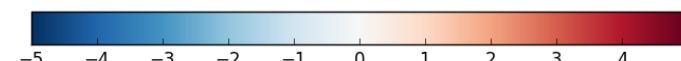
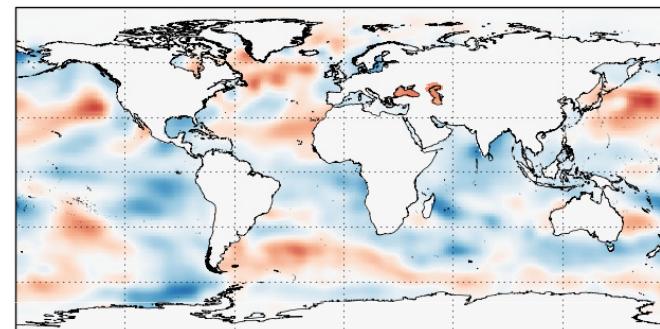


2010 data  
subsampled  
to 1960  
distribution

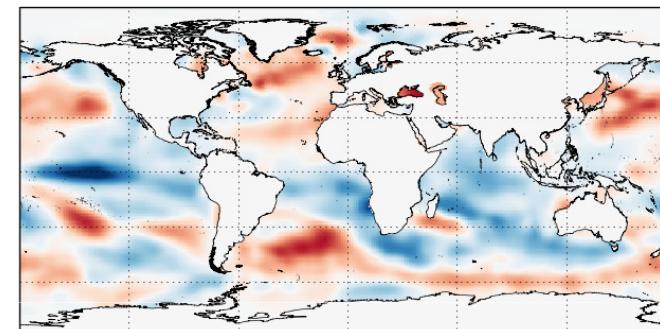
# Test results (very preliminary) ocean surface temperature increments / $^{\circ}$ C

**EOF  
DA**

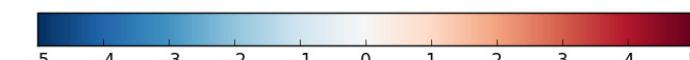
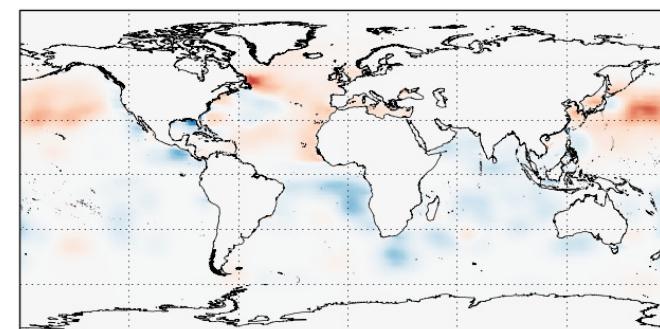
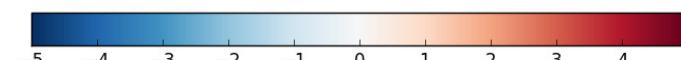
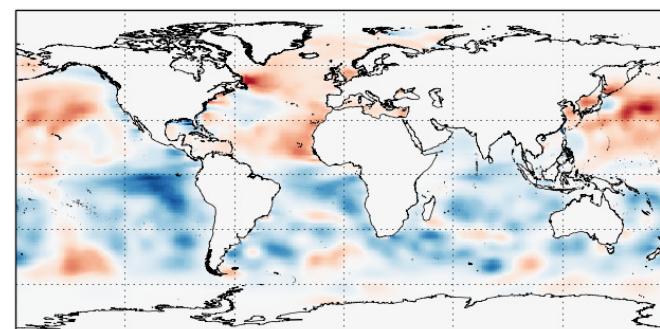
Full 2010 data



Subsampled 2010 data



**Stand  
ard  
DA**





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## Plans

- Run many observing system experiments to assess the robustness of the EOF estimates by comparing to the unassimilated data (and comparing the analysis fields)
- Compare to the standard FOAM type data assimilation (with shorter lengthscale gaussian background errors)
- Look at the impact of the source of the EOFs (model/observation based...)
- Top EOFs will likely contain useful and robust correlation structures. The lower EOFs will be less useful. Will assess how many EOFs to use to minimise the spurious increments where there are few observations (and maximise the useful increments in those locations)
- Assess multivariate & 3D aspects
- Test in a reanalysis system



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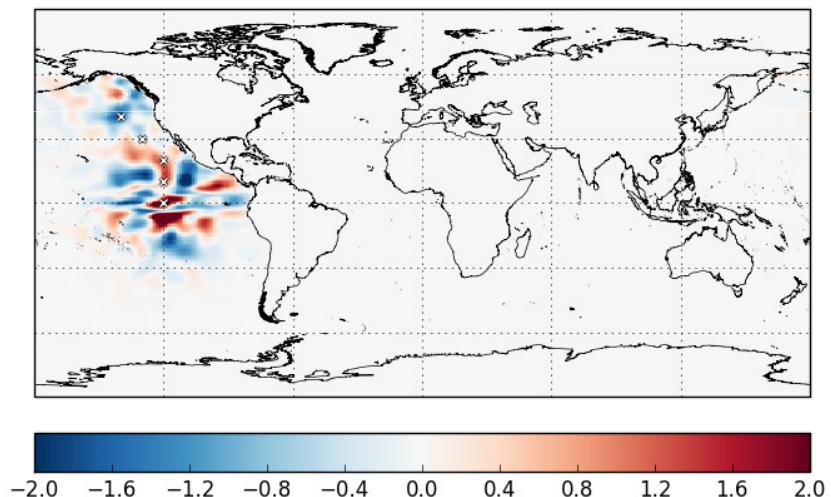
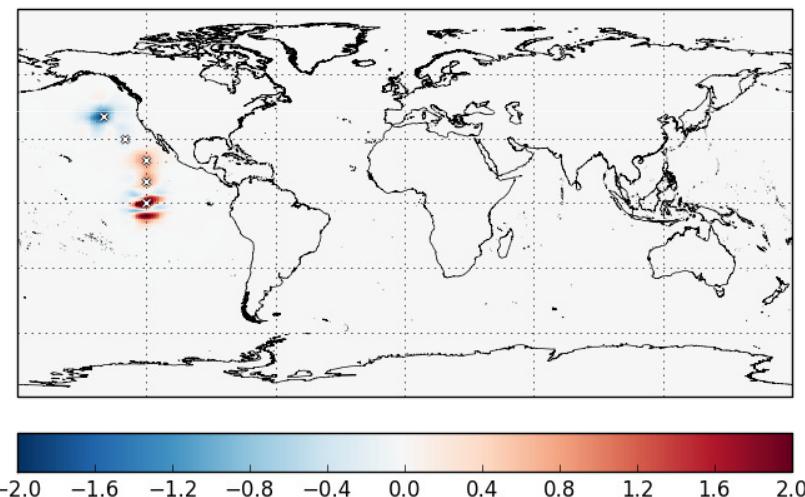
End



# Ensemble data assimilation test

Test: (random) ensemble (temperature) increments  
assimilating 5 simulated observations

Localisation uses diffusion equation (as described  
earlier)





# Diffusion model

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To spread the observation information we need to know about the covariance... – this is how much one point varies with respect to another (usually nearby point) (with a strong covariance if you make a change at one point you need to make the same change at another point).

Usually we expect close points co-vary more than distant points... (could do more complex things ... But wait until later!)

To implement this we use the diffusion equation - good for the ocean problem as it can deal nicely with coastlines:

$$\frac{du}{dt} = \nabla \cdot (K \nabla u)$$

For uniform  $K(x,y,z)=k$  for a 3D field  $u=u(x,y,z)$ :

$$\frac{du}{dt} = k \{ \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} \}$$

Explicit discretisation/timestepping (on a grid):

$$u(x,t+1) - u(x,t) = k \{ u(x+1,t) - 2u(x,t) + u(x-1,t) \} + \dots$$



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# 1D implicit diffusion

Explicit timestepping (forwards Euler)

$$u(x,t+1) - u(x,t) = k \{ u(x+1,t) - 2u(x,t) + u(x-1,t) \}$$

Implicit timestepping (backwards Euler)

$$u(x,t+1) - u(x,t) = k \{ u(x+1,t+1) - 2u(x,t+1) + u(x-1,t+1) \}$$

Rearrange and make  $u$  a vector (in space)

$$\mathbf{u}(t+1) = (\mathbf{I} - \mathbf{M})^{-1} \mathbf{u}(t)$$

This makes it stable even with long timesteps. But this is harder to solve this matrix problem (NEMOVAR v3 uses Cholesky decomposition) but it is still much cheaper than the explicit version for longer length scales relative to the grid spacing (like in orca 1/4 degree)

But coastal artifacts if not careful