Non-equivalence of parallel and serial assimilation with EnKF in presence of localisation.

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Abstract

This work draws attention to the non equivalence of parallel and serial assimilation with EnKF in presence of localisation and its possible practical implications.

Equivalence - Theory

Kalman filter:

$$\mathbf{P}^{a\,-1} = \mathbf{P}^{f\,-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}, \qquad (1)$$

where $\mathbf{P}^f$ - forecast error covariance, $\mathbf{P}^a$ - analysed error covariance; $\mathbf{H}$ - transformation matrix from observation space to state space; $\mathbf{R}$ - observation error covariance. If there are two groups of uncorrelated observations $\mathbf{O}_1$ and $\mathbf{O}_2$, so that

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{pmatrix},$$

then

$$\mathbf{R}^{-1} = \begin{pmatrix} \mathbf{R}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2^{-1} \end{pmatrix},$$

and

$$\mathbf{P}^{a\,-1} = \mathbf{P}^{f\,-1} + \mathbf{H}_1^T\mathbf{R}_1^{-1}\mathbf{H}_1 + \mathbf{H}_2^T\mathbf{R}_2^{-1}\mathbf{H}_2. \qquad (2)$$

This means that $\mathbf{O}_1$ and $\mathbf{O}_2$ can be assimilated in arbitrary order or simultaneously, resulting in the same analysed covariance. This fact is often referred to as equivalence of parallel and serial assimilation (EPSA).

Implications of the Equivalence

- EPSA is widely used in data assimilation systems by grouping assimilated data in an operationally convenient way [1].
- The square root scheme by Potter is based on EPSA [2].

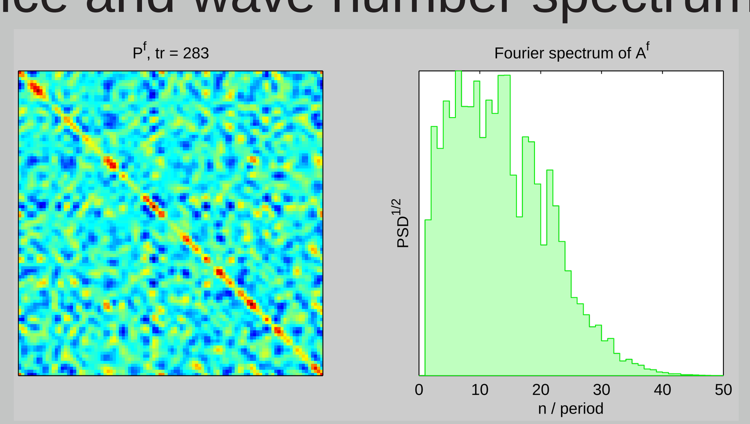
Suggestion

EPSA is only valid in the strict Kalman filter framework, and any ad-hoc modifications of the filter can destroy it. In particular, it may no longer be valid in presence of localisation.

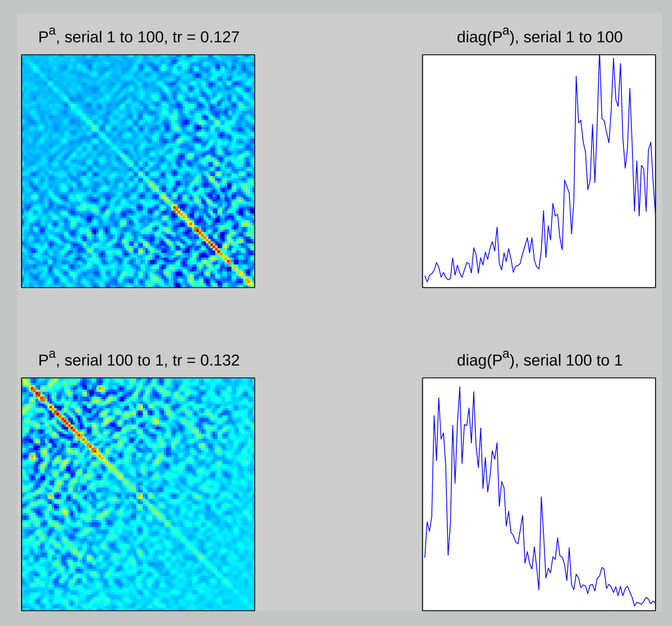
Experiment 1: serial versus parallel

To confirm the above suggestion, we conduct the following experiment:

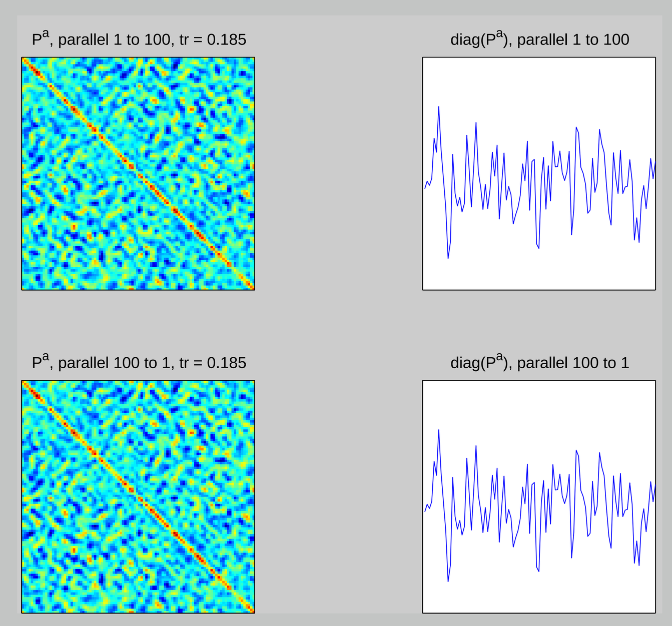
- Setup: domain = 1D, periodic; state vector dimension $\boldsymbol{n} = \mathbf{100}$; ensemble size $\boldsymbol{m} = \mathbf{10}$; number of observations $\boldsymbol{p} = \mathbf{100}$; observation error variance $\boldsymbol{\sigma}^2_{obs} = \mathbf{0.01}$; localisation radius $\boldsymbol{r}_{loc} = \mathbf{20}$; localisation function $\boldsymbol{f} = \exp(-\boldsymbol{r}^2/\boldsymbol{r}^2_{loc})$; localisation method = covariance filtering.
- Generate ensemble with the following forecast covariance and wave number spectrum:



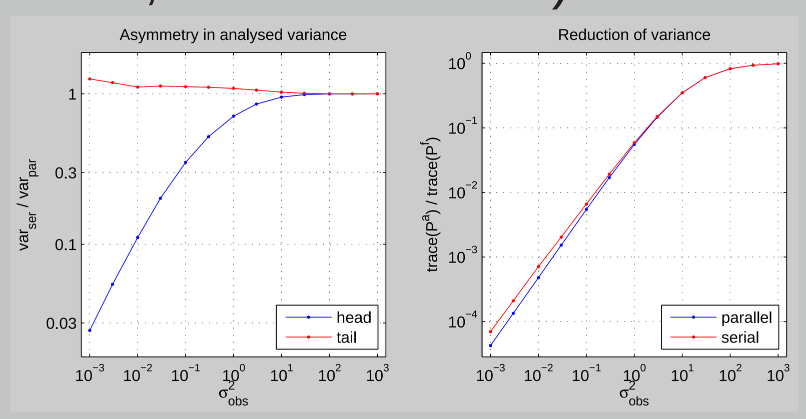
- Assimilate serially twice: (i) from element 1 to element 100 and (ii) from element 100 to element 1. The results are as follows:



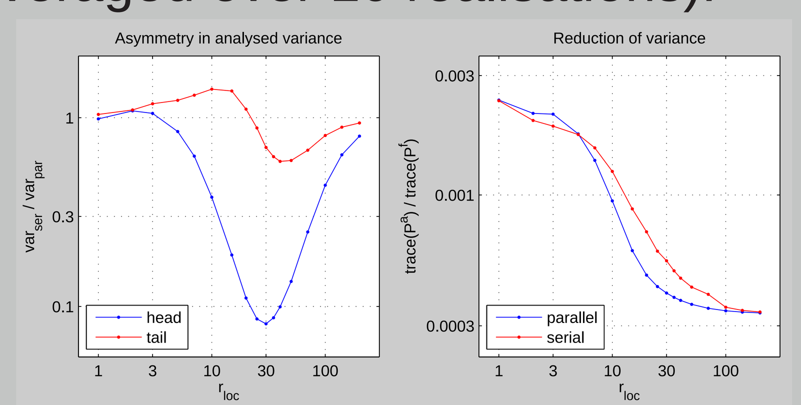
- Assimilate in parallel twice: (i) $\mathbf{H} = \mathbf{eye}(\boldsymbol{n})$ and (ii) $\mathbf{H} = \mathbf{fliplr}(\mathbf{eye}(\boldsymbol{n}))$, using left-multiplied ESRF: $\mathbf{A}^a = (\mathbf{I} + \rho \circ \mathbf{P}^f\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^{-1/2}\mathbf{A}^f$. The results are as follows:



- Compare the analysed covariance of serial and parallel assimilation for different assimilation strengths (averaged over 10 realisations; "head" corresponds to first 10 observations, "tail" - to last 10):



- Compare the analysed covariance of serial and parallel assimilation for different localisation radii (averaged over 10 realisations):



Note: The above results can be reproduced with a different localisation method (scaling of observation variance) or localisation function (e.g. with Gaspari & Cohn polynomial).

Conclusions from Experiment 1

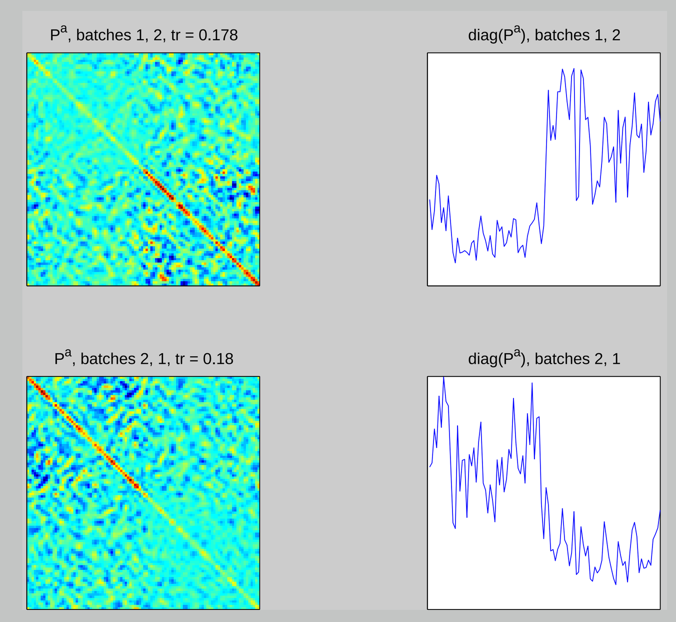
- Localisation breaks up EPSA.
- The effect is larger for stronger assimilation.
- For the effect to be substantial the assimilation needs to be fairly strong (variance reduction by an order or more).
- Serial processing with localisation results in stronger reduction of variance for data assimilated first.
- Asymmetry in analysed covariance reaches maximum at some intermediate localisation radius.

Question

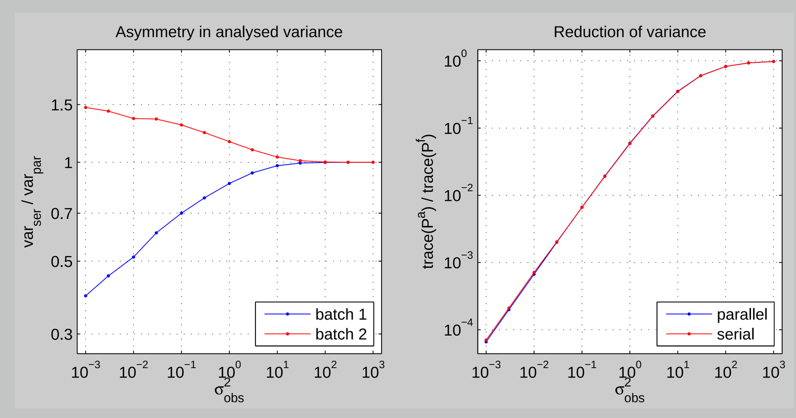
Would assimilating in bigger batches of observations reduce the asymmetry in analysed covariance?

Experiment 2: in batches versus parallel

Set as Experiment 1 except that serial assimilation is now performed with two batches of data, of 50 consecutive observations in each. We assimilate serially twice: (i) batch 1, then batch 2 and (ii) batch 2, then batch 1. The results are as follows:



Compare the analysed covariance of serial and parallel assimilation of big batches of data for different assimilation strengths (averaged over 10 realisations:



Conclusion from Experiment 2

- Assimilating in bigger batches of observations reduces the asymmetry in analysed covariance, but it still can be noticeable.

Conclusions

- Localisation breaks up EPSA.
- The effect reduces when assimilating observations in big batches.
- The effect is substantial for very strong assimilation only and therefore should not be of major concern for practical DA systems.

Literature

[1] P. L. Houtekamer and H. L. Mitchell. A sequential ensemble kalman filter for atmospheric data assimilation. *Mon. Wea. Rev.*, 129:123–137, January 2001.

[2] J. S. Whitaker and T. M. Hamill. Ensemble data assimilatrion without perturbed observations. *Mon. Wea. Rev.*, 130:1913–1924, July 2002.

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