An ensemble prediction study of the East Australian Current

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Introduction:

The perturbation dynamics in extended chaotic systems may be examined using the general method for the calculation of Lyapunov vectors. Using this approach one may generate initial perturbation vectors that contain by construction different types of information about the chaotic trajectory. The type of information encoded is dependent on the initial perturbation magnitude (infinitesimal or finite) and the evolution interval for the calculation (finite or quasi-infinite; past, future or both).

In preparing an ensemble of initial perturbations there are a number of properties that are desirable. The four most outstanding properties usually required are the following:

(i) Dynamic adaptation: initial perturbations should be well embedded in the attractor, this can be achieved by growing perturbations from the (remote) past.

(ii) Reliability and equivalence among the members of the ensemble: One wants the perturbations to be statistically equivalent but at the same time to have enough diversity to capture a significant portion of the phase space.

(iii) Analysis of error: the ensemble should be able to capture differences between the analysis and the true state.

(iv) Fastest spread: The ensemble should be able to sample the fastest growth directions in phase space, so that the more unstable directions are well represented.

Dynamically adapted vectors from the past such as BVs are then good candidates to generate the desired ensemble (Pazo et al 2010). Corazza et al. (2003) presented results suggesting that a systems bred vectors have similar structures to the systems ‘error of the day’ (QG, synthetic obs, no model error, etc). Predictability of large scale regime transitions are sensitive to small scale error growth and the spatial structures of small and even subgrid errors (O’Kane & Frederiksen 2008, Nadiga & O’Kane 2010).

Open questions:

Does this hold for an operational ocean system?

If so how many perturbation vectors are required to identify when and where instabilities are likely to occur in the ocean; and so-doing identify when and where the forecast skill of an operational ocean forecast system is likely to be low?

How do typical errors for the EAC (OceanMAPS analysis, MOM4p1) grow (linearly or nonlinearly)?

Can a BV ensemble provide information useful for adaptive sampling?

Could BVs be a useful method to introduce flow dependent information into a static background error covariance matrix (hybrid DA-EPS)? Errors of the day.

Ensemble forecasting

The use of ensemble forecasting and data assimilation in NWP has made apparent the importance of local spatio-temporal predictability properties of the atmosphere in space and in time (e.g., Toth and Kalnay, 1993, Molteni and Palmer, 1993).

The “spaghetti” plots and other methods used to display operational ensemble products frequently show simultaneous high predictability in some areas and low predictability in others.

The regional loss of predictability is an indication of the instability of the underlying flow, where small errors in the initial conditions (or imperfections in the model) grow to large amplitudes in finite times.

The stability properties of evolving flows have been studied using Lyapunov vectors (e.g., Alligood et al, 1996, Ott, 1993, Kalnay, 2001), singular vectors (e.g., Lorenz, 1965, Farrell, 1988, Molteni and Palmer, 1993), and, more recently, with bred vectors (e.g., Szunyogh et al, 1997, Cai et al, 2001, O’Kane & Frederiksen 2008).

Lyapunov vectors and singular vectors• Lyapunov vectors: fundamental (invariant) time-dependent normal modes for instability of time-dependent flow• Singular vectors: optimal transient growth in specified norm over fixed time interval

Bred vectors (BVs) are, by construction, closely related to Lyapunov vectors (LVs). In fact, after an infinitely long breeding time, and with the use of infinitesimal amplitudes, bred vectors are *identical to leading Lyapunov vectors.*

In practical applications, however, bred vectors are different from Lyapunov vectors in two important ways: a) bred vectors are never globally orthogonalized and are intrinsically local in space and time, and b) they are finite amplitude, finite time vectors. i.e. nonlinear generalization of a LV

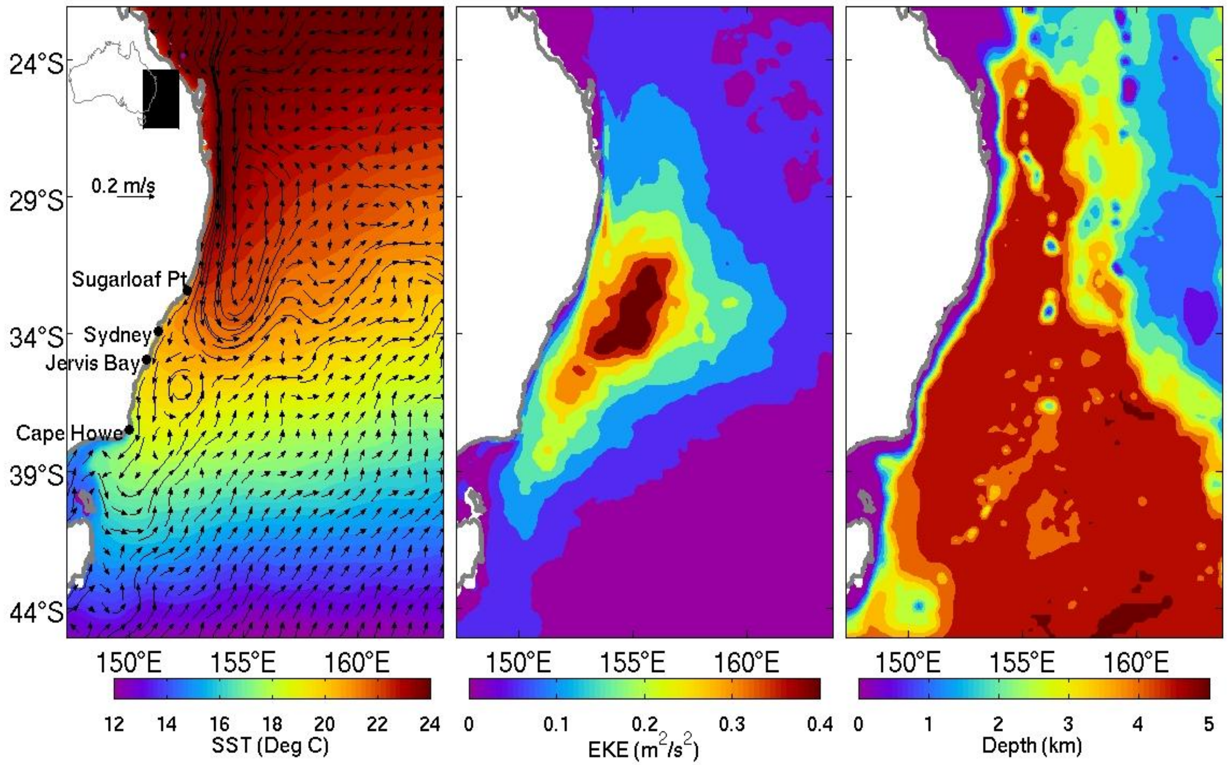
These two differences are very significant in a very large dynamical system. For example, the ocean/atmosphere is large enough to have “room” for several mesoscale instabilities to develop independently in different regions, and it is complex enough to have different possible types of instabilities (such as barotropic, baroclinic, normal, non normal, linear, nonlinear, homogeneous, inhomogeneous).

BVs can capture an asymmetric nonlinear response to a finite amplitude perturbation

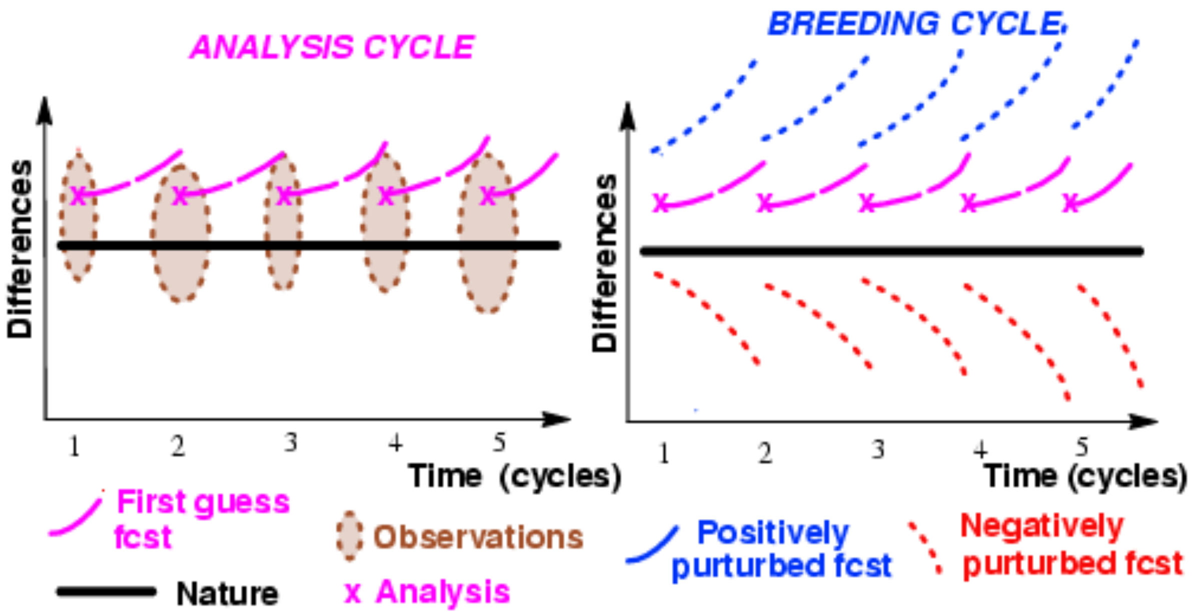
East Australian Current

- Part of the Western Boundary Current (WBC) system associated with the South Pacific subtropical gyre, the EAC forms near 15◦S, and flows along the coast carrying on average 22 Sv attaining its maximum volume transport at 30◦S then tending to separate from the coast near Sugarloaf Point (32:5◦S) before flowing southeastward into the Tasman Sea. After the EAC separates from the coast it spawns a rich field of mesoscale eddies that are evident in the time-mean eddy-kinetic-energy field. The EAC is complex and characterized by large seasonal and mesoscale variability, wind driven upwelling and eddy formation and strong eddy-eddy, eddy-mean and eddy-topographic interactions.
- Warm-core eddies are typically large, with diameters of several hundred kilometers, forming every 90 days or so (Mata et al. (2006)). Cold-core eddies are smaller, perhaps 50-100 km across (e.g. Oke and Griffin (2010)), and often form at the point where the EAC separates from the coast, or on the peripheries of warm-core eddies. While warm-core eddies are usually well-resolved by altimetry, cold-core eddies are often missed.

Thirteen-year average (1993-2006) SST and surface velocities (left), eddy-kinetic energy (EKE; middle) computed from daily mean fields of surface velocity from BRAN2p1 Schiller et al. (2008), and model topography (right). The inset on the left panel shows the location of the region of interest off south eastern Australia.



Bred perturbation vectors

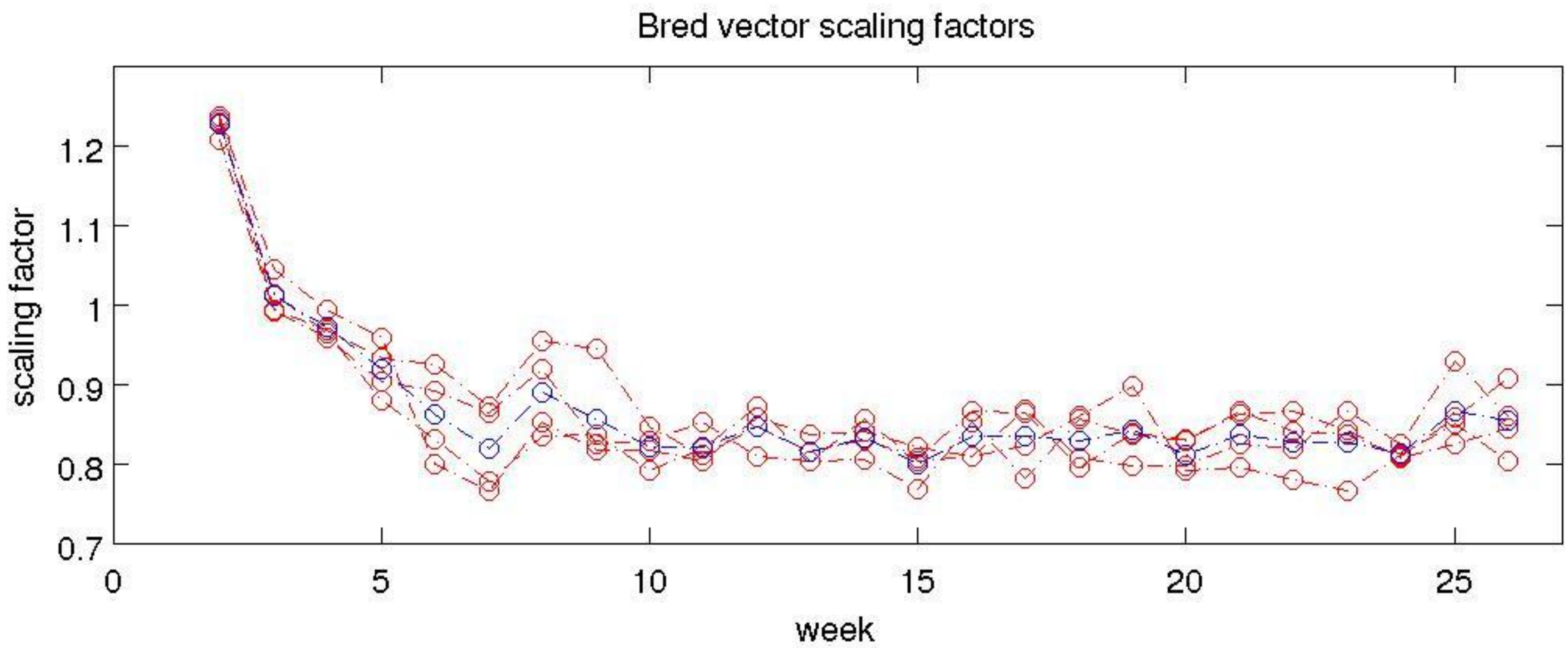


- Random isotropic initial perturbations grow more slowly and lead to underestimated error variances.
- In EP independent initial disturbances are generated as fast growing disturbances with structures and growth rates typical of the analysis errors.
- Analysis cycle acts as a nonlinear perturbation model on the evolution of the real flow resulting in error growth associated with the evolving state to develop within the analysis cycle and dominate forecast error growth.
- We would like to generate independently perturbed initial conditions such that the covariance of the ensemble perturbations ≈ initial analysis error covariance at the time of the forecast.
- Breeding method forecast perturbations are transformed into analysis perturbations in order to sample the subspace of most rapidly growing analysis errors
- The choice of rescaling amplitude and rescaling interval determines the instability that the perturbation vectors project onto

Model and Data Assimilation

- Analysis is provided by OceanMAPS which is comprised of the Ocean Forecasting Australian Model (OFAM), based on the GFDL MOM4 code (Griffies et al.(2003)), with 1=10◦ resolution in the region 90◦E-180◦E; 75◦S-16◦N with decreasing resolution elsewhere.
- OFAM is initialized by the Bluelink Ocean Data Assimilation System (BODAS; Oke et al. (2008)) based on ensemble optimal interpolation (EnOI) with background error covariances defined from a time invariant ensemble of seasonal anomalies (72 members at present) derived from a long model integration without data assimilation.
- OceanMAPS uses a 7 day window of observations that include SLA from altimetry, temperature & salinity profile observations from XBT, CTD and Argo, and AMSRE SST.
- The model we use is a regional version of OFAM. It also has 0.1◦ horizontal resolution and 47 z-levels with 10m vertical resolution in the upper 200m, expanding to larger spacing towards full oceanic depths. The regional model is nested inside the global model and uses sponge layer open boundary conditions for surface height , temperature T, salinity S, and the u and v components of the velocity field, and uses the same grid and bathymetry as the global model. The domain chosen for this study covers an area in the Tasman Sea enclosed within 148:75◦E to 163:75◦E and 45:05◦S to 22:05◦S
- The regional model is forced by surface fluxes from the analysis NWP cycle that is run operationally by the Australian Bureau of Meteorology. An adaptive dynamical initialization scheme (Sandery et al. (2010)) that applies time and space dependent tendency forcing that is a function of differences between model and target prognostic variables.

week



“spun up” BVs are independent of type of initial perturbation, isotropic random, anomalies, heave?.

Rescaling amplitude and evolution window are critical.

u, v, salinity, temp, eta all rescaled against error growth at T250

Surface height. Columns 1, 2, 3 & 4 depict results valid for the 4th, 11th, 18th & 25th March 2008 respectively.

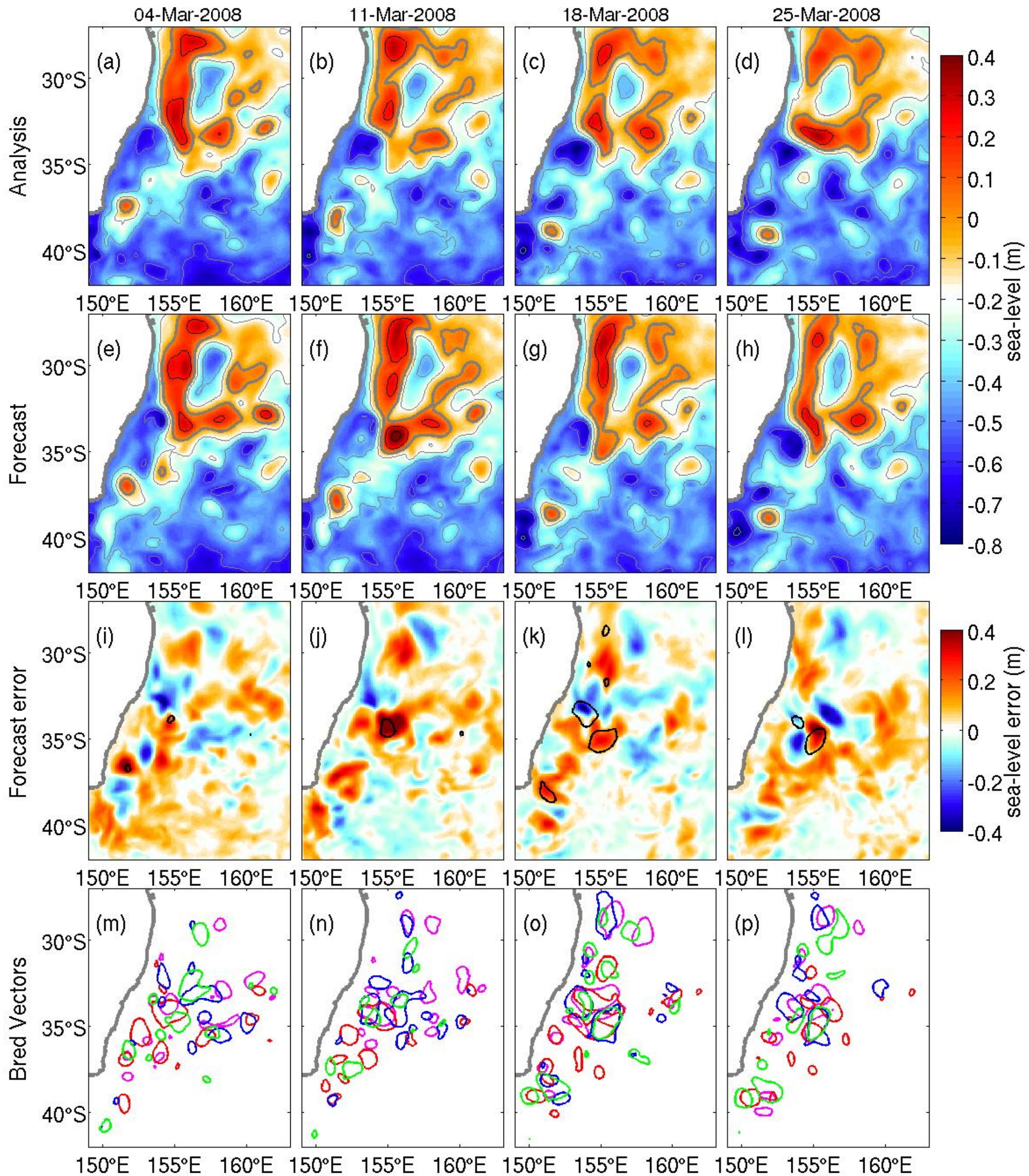
Row 1 figures a, b, c & d; Analysis fields,

Row 2 figures e, f, g & h; 7 day control forecasts.

Row 3 figures i, j, k & l; Comparison of ensemble averaged (4 members) bred vectors (0:35m contours) and day 7 forecast error (shaded) valid for the I) 4th; j) 11th; k) 18th & l) 25th March 2008.

Row 4 figures m, n, o, & p; 0:35m contours for each of the 4 individual bred vectors.

Note: there is a high degree of diversity amongst the individual BVs making them suitable for direct use as members of the ensemble. This is typically NOT the case for atmospheric flows where BVs grow on a large scale basic state requiring either orthogonalization or alternate approaches to increasing spread before being viable ensemble members. Also infinitesimal amplitude perturbations (linear error growth; SVs, LVs) are unsuitable due to strength of nonlinearity.



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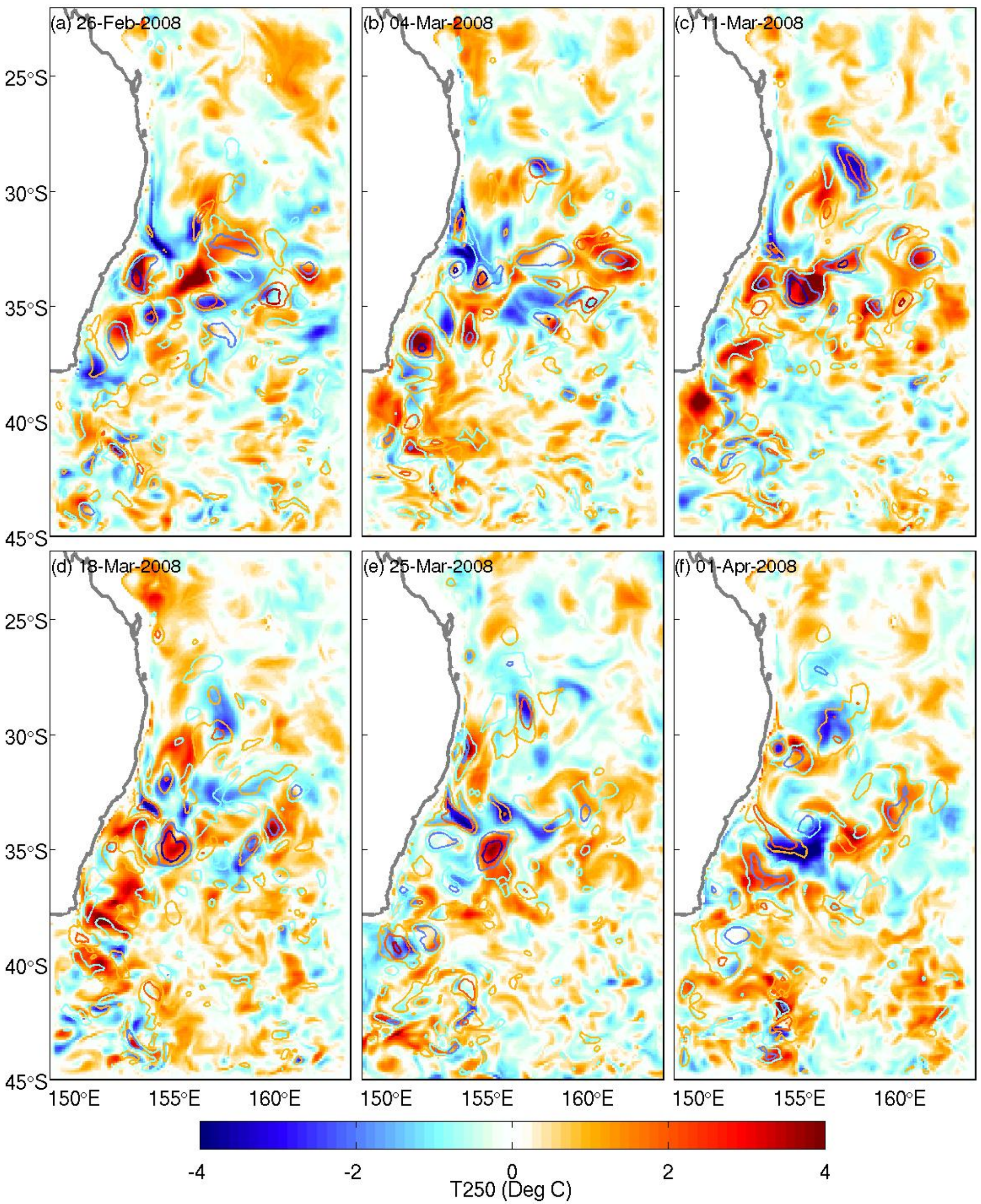

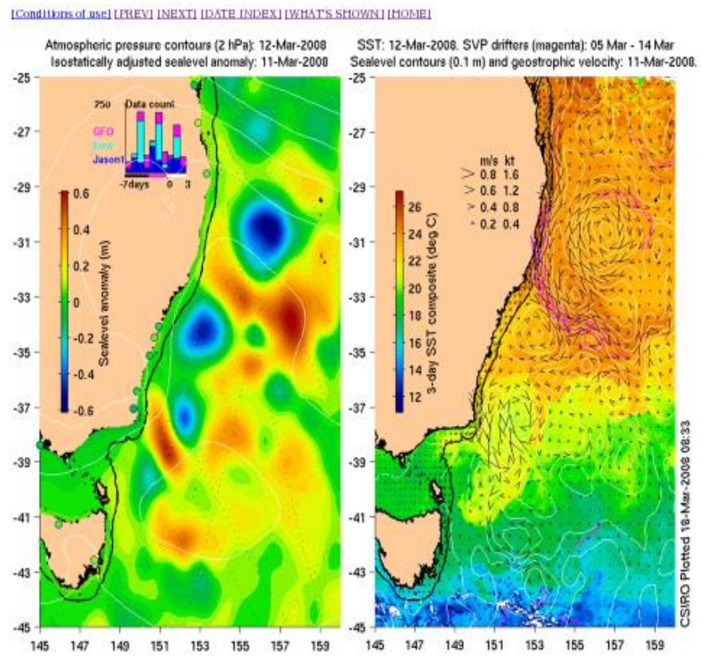

Comparison of ensemble averaged evolved bred vectors (4 members contours) and forecast error (shaded) for T250 at weekly intervals over a 6 week period beginning on the 26th February 2008. Contours shown are 3:5◦C, 2:0◦C & 1:0◦C.

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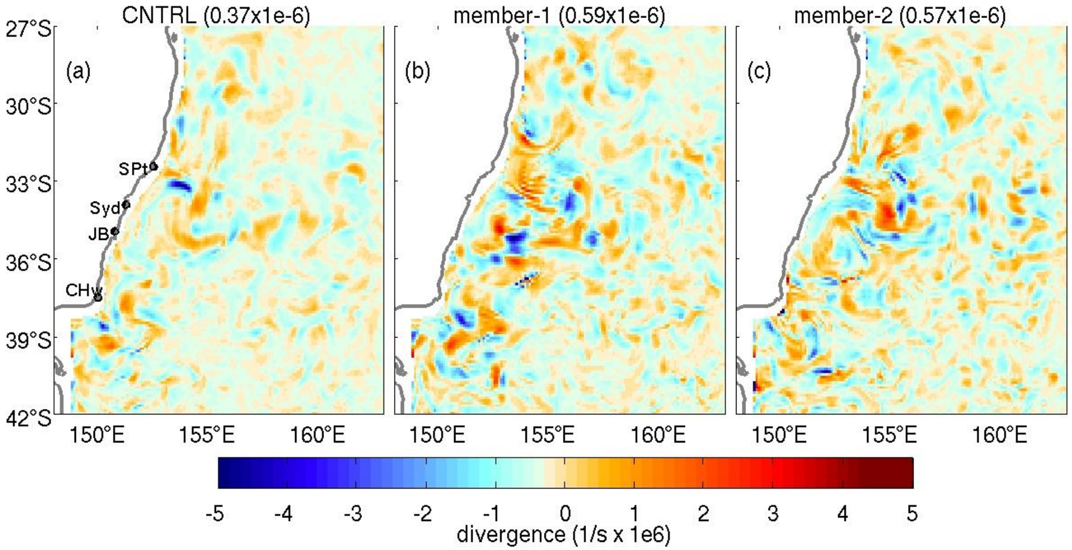
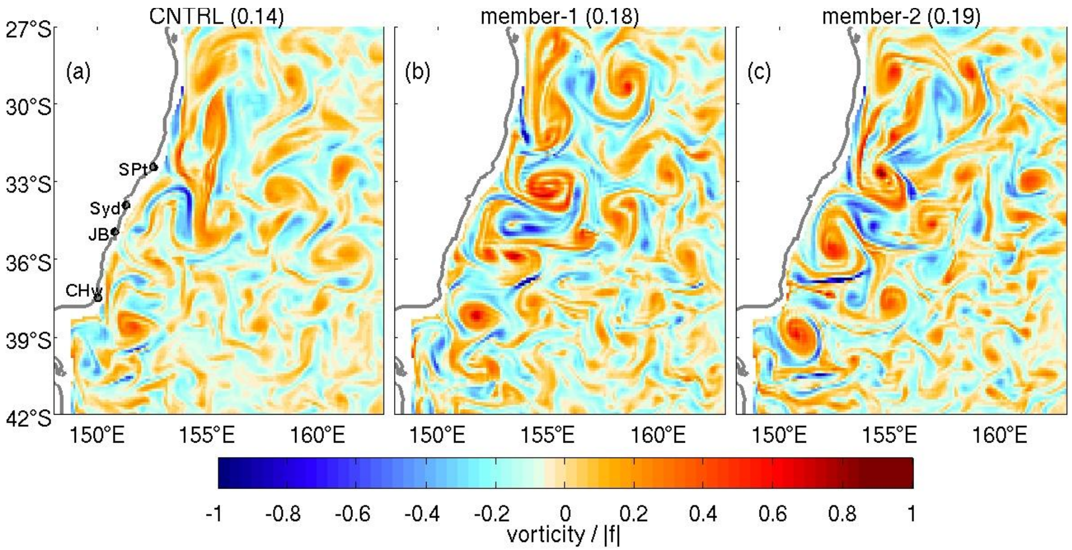
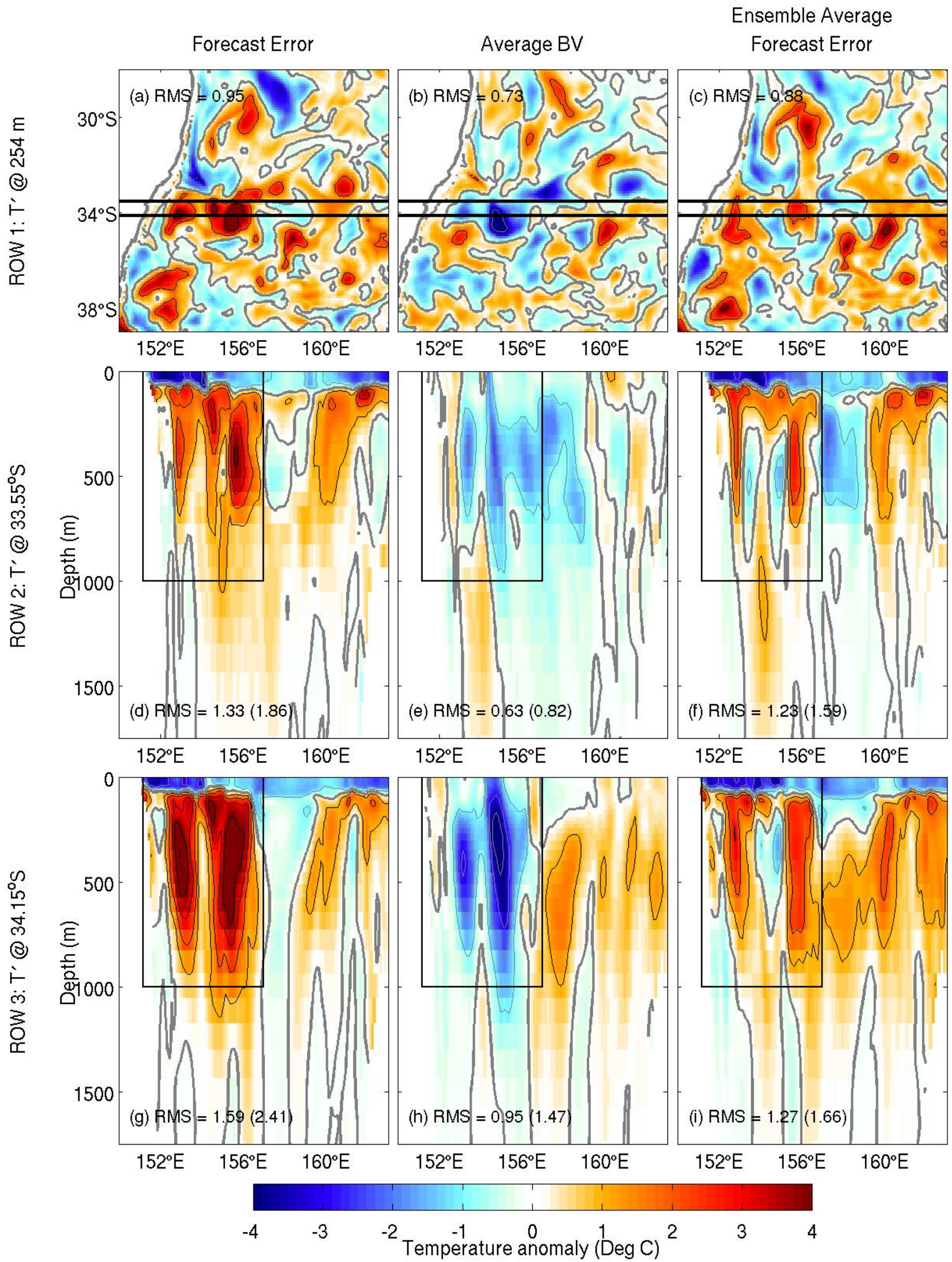
Area averaged Rossby number (depth averaged 0-200m relative vorticity divided by coriolis parameter) and corresponding divergence for 7 day forecasts valid for the 18th March 2008. Figure a) control forecast, b) & c) individual perturbed forecasts.

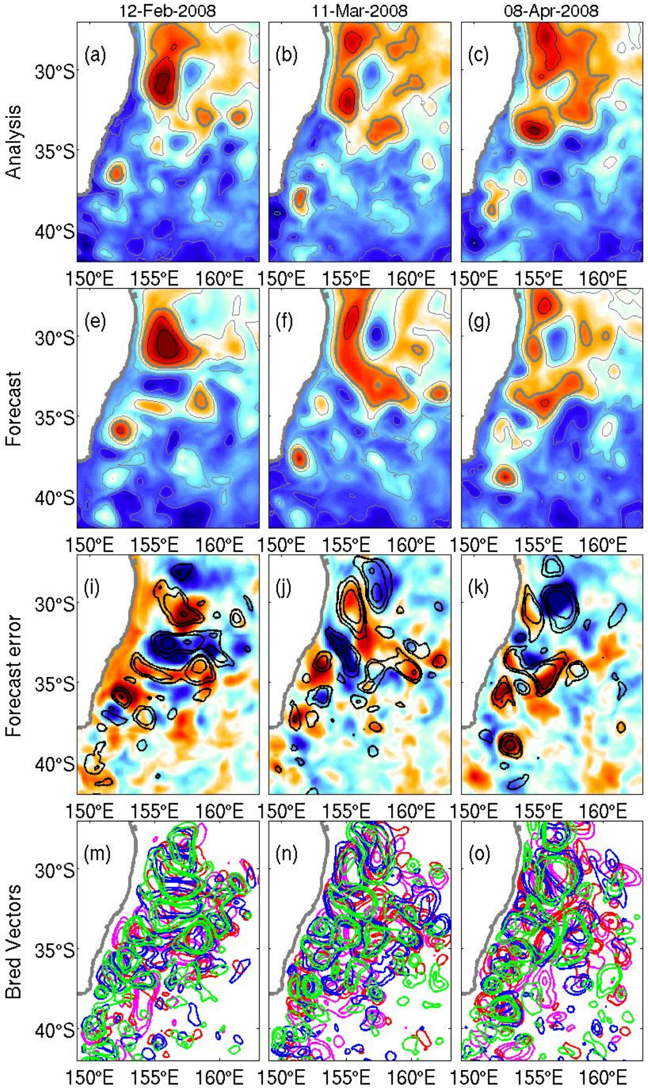


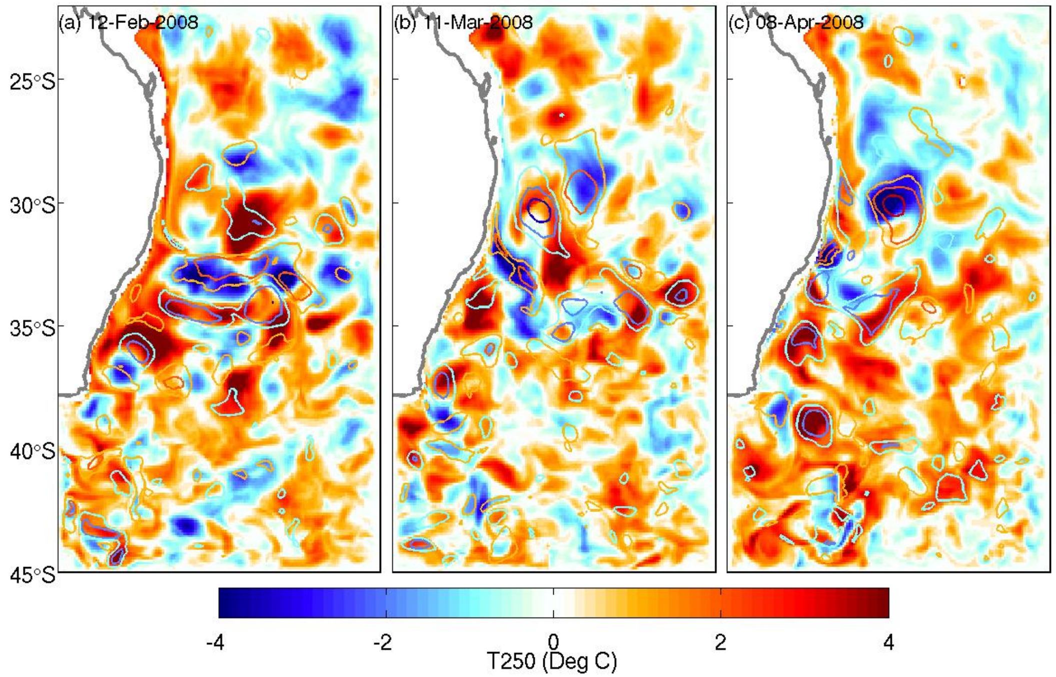


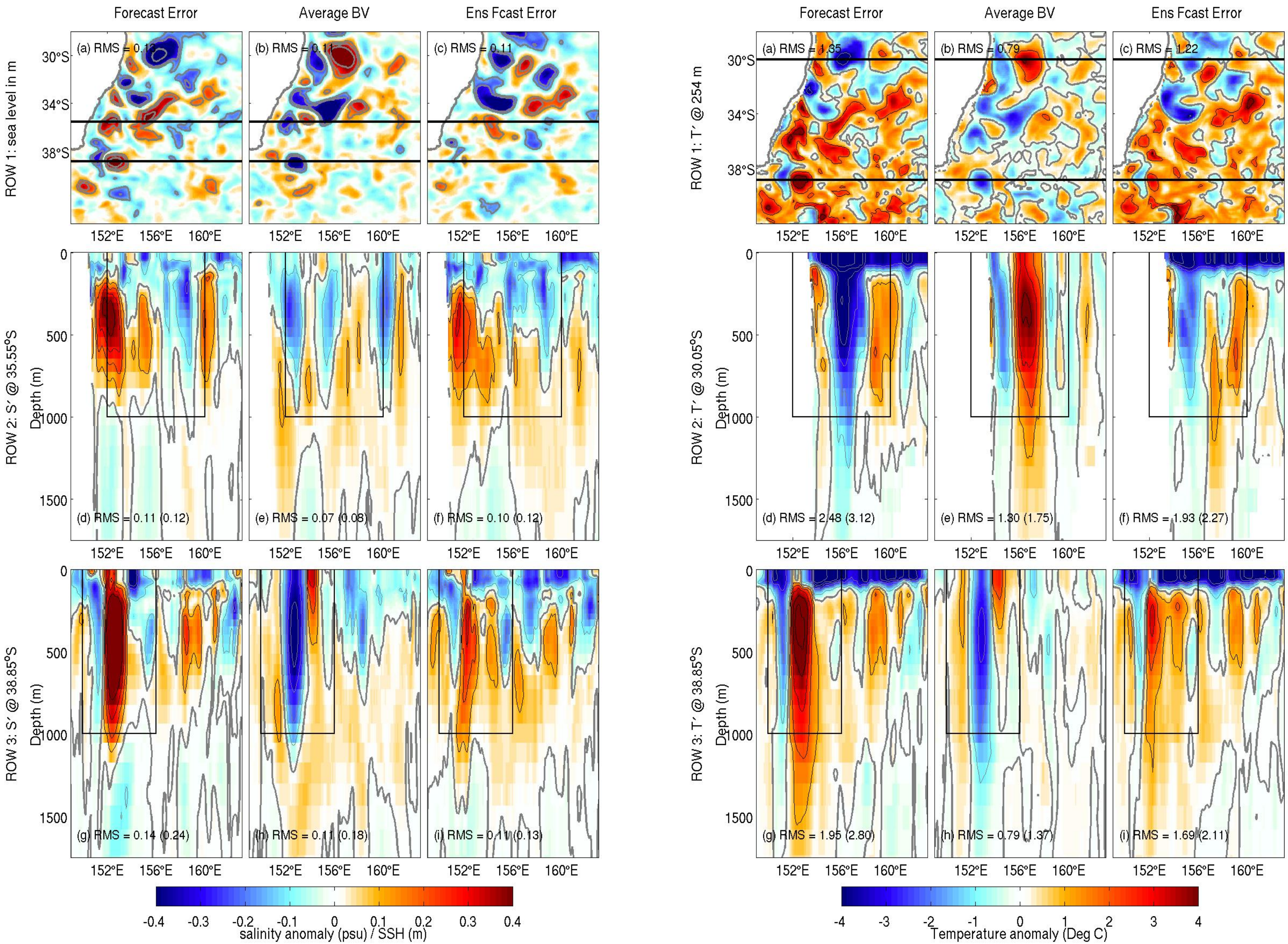
Comparison of day 7 forecast error (left), ensemble averaged bred vectors (middle) & ensemble averaged forecast error (right) (4 members) valid on the 11th March; Showing T250 (a-c) and vertical sections



1 month forecast/rescaling



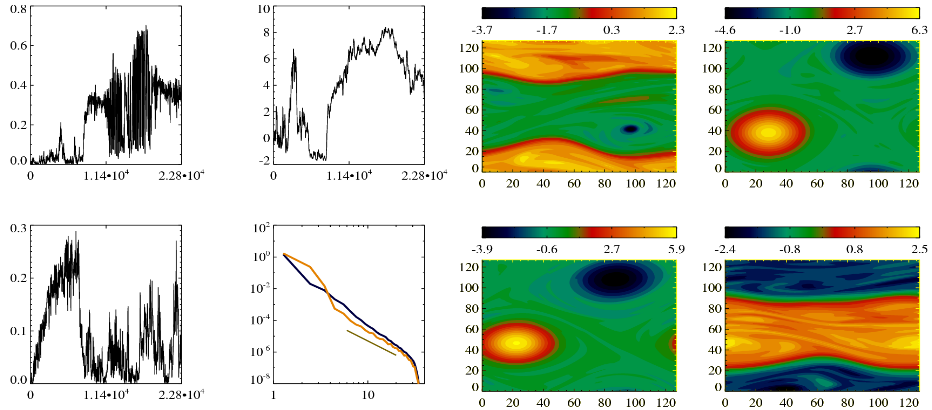



Forecast Error
Average BV
Ens Fcast Error
ROW 1: sea level in m
(a) RMS = 0.12
(b) RMS = 0.11
(c) RMS = 0.11
30°S
34°S
38°S
152°E
156°E
160°E
ROW 2: S′ @ 35.55°S
Depth (m)
0
500
1000
1500
(d) RMS = 0.11 (0.12)
(e) RMS = 0.07 (0.08)
(f) RMS = 0.10 (0.12)
ROW 3: S′ @ 38.85°S
(g) RMS = 0.14 (0.24)
(h) RMS = 0.11 (0.18)
(i) RMS = 0.11 (0.13)
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
salinity anomaly (psu) / SSH (m)
Forecast Error
Average BV
Ens Fcast Error
ROW 1: T′ @ 254 m
(a) RMS = 1.35
(b) RMS = 0.79
(c) RMS = 1.22
ROW 2: T′ @ 30.05°S
(d) RMS = 2.48 (3.12)
(e) RMS = 1.30 (1.75)
(f) RMS = 1.93 (2.27)
ROW 3: T′ @ 38.85°S
(g) RMS = 1.95 (2.80)
(h) RMS = 0.79 (1.37)
(i) RMS = 1.69 (2.11)
-4
-3
-2
-1
0
1
2
3
4
Temperature anomaly (Deg C)

Conclusions

- We have conducted an ensemble prediction study of the EAC with a specific focus on the examination of the role of dynamical instabilities and flow dependent errors of the day.
- Our EPS is based on the breeding method to identify the fast growing errors of the day for a given initial state. Here the initial state is an analysis product from the Australian operational ocean forecast system which employs an EnOI DA scheme.
- We considered a period spanning the Austral summer and autumn of 2008 corresponding to the seasons of largest eddy variability.
- We have shown that individual perturbations generated as bred vectors, while globally distinct, collapse onto a low dimensional subspace that correspond to regions of large dynamic instability where forecast errors are large and in particular where the EAC separates from the coast.
- Our results clearly show that over 7-30 days forecast errors arise due to dynamic instabilities and that these forecast errors can be expected to dominate analysis errors. Moreover BVs are a promising and computationally inexpensive means to calculate flow dependent background information critical to accurate forecasting.
- Further a very small BV ensemble may be run to identify regions where additional observations may be targeted.
- A generalization of the method is a useful tool for identifying mechanisms for regime transitions.

LFV and predictability of the Stochastic Navier-Stokes Equation (F-plane)



$$\frac{D\omega}{Dt} = \frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \mathbf{F} + \mathbf{D}$$

velocity is given by $\mathbf{u} = \mathbf{e_z} \times \nabla \psi$ and $\omega = \nabla^2 \psi$

$$Domain = 2\pi \times 2\pi\delta;\ \delta = 1.1$$

$D = -\alpha\omega + \nu\triangle\omega$where α is a frictional constant and $\nu\triangle\omega$ is a small-scale dissipation term implemented as a high pass filter of the vorticity field with a characteristic wavenumber $k_d \approx 0.61 k_{max}$

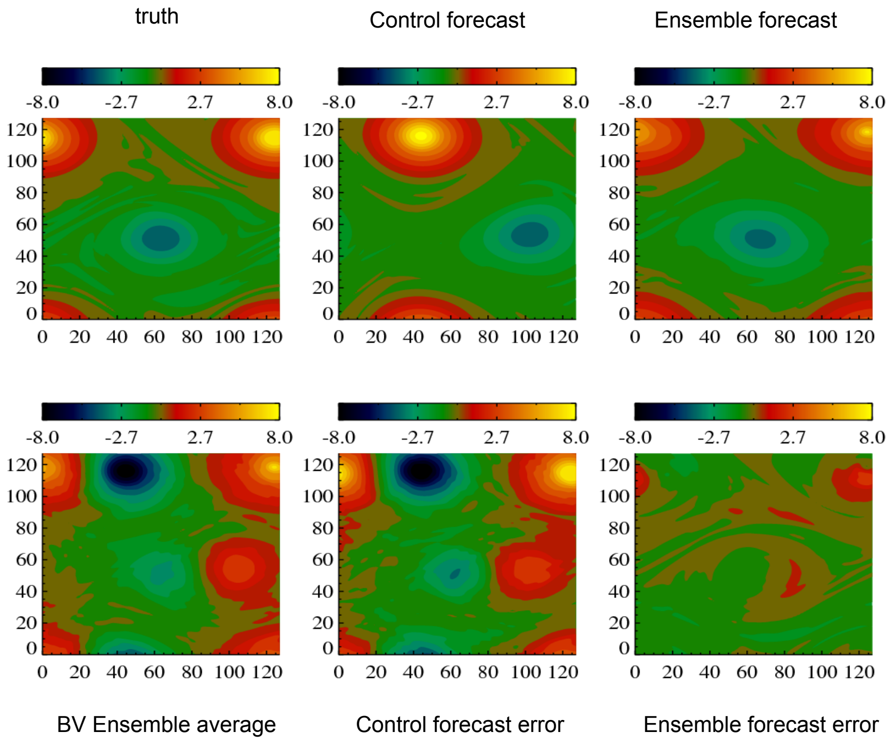
$F = \sqrt{2\alpha}\tilde{F}$ where $\tilde{F}$ is isotropic forcing at characteristic wavenumber $k_f \approx 3$ and $\langle \tilde{F}_{\mathbf{k}}(t)\tilde{F}_{\mathbf{k}'}(t')\rangle = \delta_{\mathbf{k}\mathbf{k}'}(t-t')$

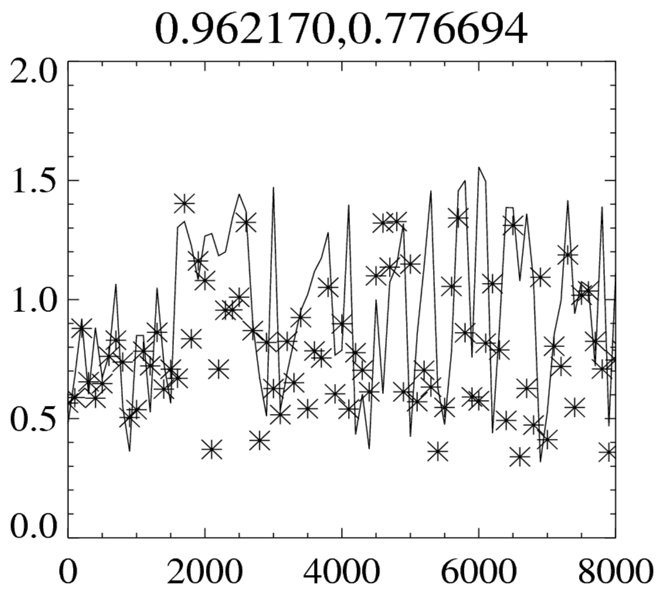
$\alpha = 10^{-4}$ leading to a dissipation timescale of 10^4 eddy turnover times

i.e. energy input due to forcing is $O(10^4)$ smaller than nonlinear flux of energy in the system

The weakness of the linear damping results in a damping scale for energy that is larger than the domain size allowing energy to condense onto the largest scales. In this regime the model displays behaviour that is somewhat analogous to a subcritical pitchfork bifircation in the presence of noise i.e. under the influence of weak random forcing the model switches randomly and abruptly between zonal and dipolar states.

Predictability of SNS (zonal to dipolar)



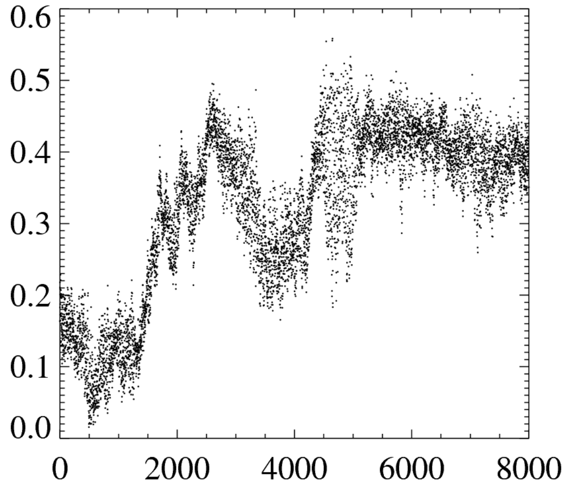


Control forecast error (line) versus ensemble forecast error (stars) at lead time of 200 eddy turnover times. To: average RMS errors.

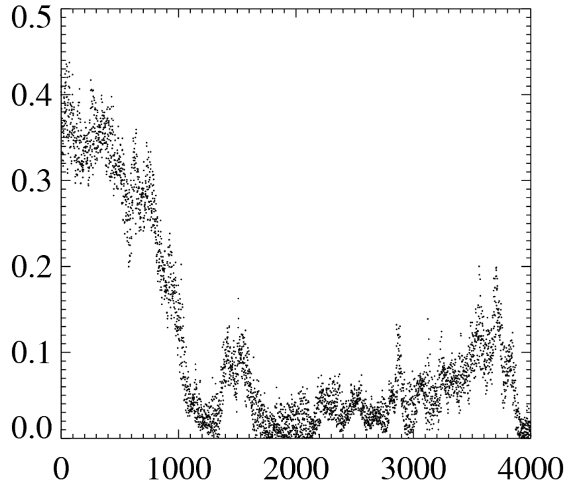
We first generate a truth trajectory as a long run of the model over at least one regime transition. In order to mimic a simple data assimilation scheme, the large scales (1 < k < 5) of both control and perturbed trajectories are nudged every 200 eddy turnover times to the true state, with the strength of nudging set so that control forecast error (defined at the forecast time as the standard deviation of the difference between the truth and control) was comparable on average to the standard deviation of the truth field. A small forecast ensemble (control plus 6 perturbed members) is constructed about the control analysis using bred perturbation vectors [8, 9]. Every rescaling period (chosen to be about 10% of the transition time or 100 eddy turnover times in the present model) the vector difference between the perturbed and control runs are rescaled to the initial amplitude (10% of the standard deviation of the background or natural variability) using the RMS-norm and added to the control run.

Regime transitions

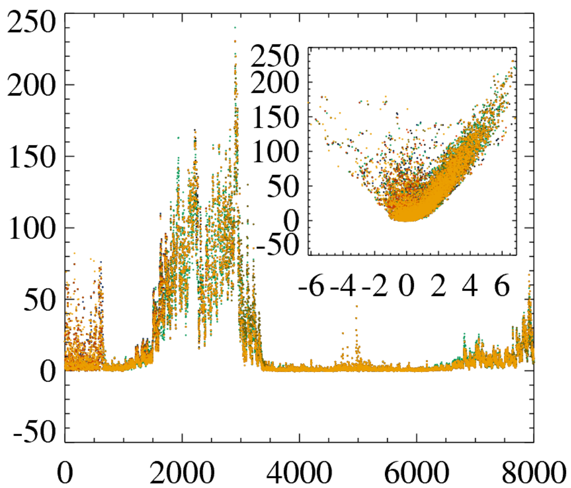
Zonal-dipolar

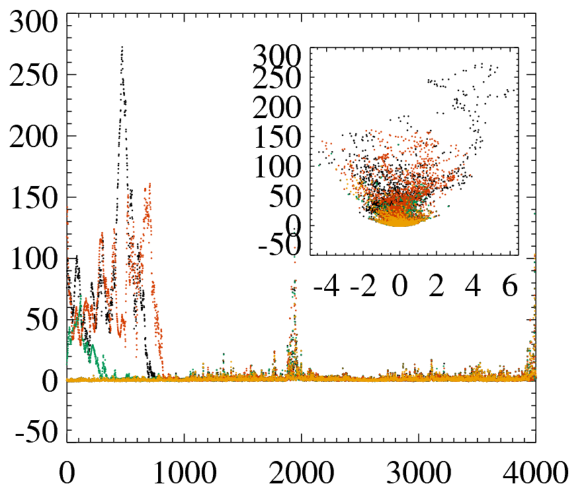


Dipolar-zonal



Energy in zonal wavenumber 1versus eddy turnover time





Kurtosis of perturbation vectors. (inset) Skewness against kurtosis

Transitions are characterized by strong response at the small scales. Small scale error growth is dominated by non-Gaussian terms indicative of Palinstrophy production. Individual bred perturbation vectors form in regions of large shear growing on coherent structures.
Introduction of stochastic "backscatter" at the small scales inhibits the rapid amplification of these higher order effects.
We postulate that deterministic backscatter is required.

Because of their ease of construction, knowledge of the past (flow dependency) and dynamical balance we have chosen to characterize space–time chaos in the EAC using bred vectors.

BVs are finite perturbations generated (or ‘bred’) by imposing that the perturbed system $\omega'(t)=\omega(t)+\delta\omega(t)$ stays within some fixed finite distance from the control trajectory $\omega(t)$.

This is simply achieved by periodically rescaling the error to a given size ε dependent on a time interval T.

The difference between control and perturbed trajectories $\delta\omega(t+\delta t) =\omega'(t+\delta t)-\omega(t+\delta t)$ is computed at times $\delta t=nT$ for n 1,...,N where upon the evolved perturbation is rescaled and the perturbed system redefined as $\omega'(t+\delta t)=\omega(t+\delta t)+\varepsilon\delta\omega(t+\delta t)/||\delta\omega(t+\delta t)||$.

The system is now allowed to evolve freely until the next rescaling is scheduled at time (n+1)T.

The bred vector corresponds to the (finite) error $\delta\omega(t)$ constructed at time t.

The time interval T is chosen to be smaller than the average error saturation time.

Computation of Lyapunov vectors

The leading Lyapunov vector is computed as follows:1) Start with an arbitrary perturbation of arbitrary size2) Evolve it from to using the TLM 3) Repeat 2) for the succeeding time intervalsAfter a sufficiently long time , the perturbation converges to the leading Lyapunov vector. The direction of this vector is independent of the initial perturbation, the length of the time interval or the choice of norm of the perturbation, properties not shared by singular vectors. If during the repeated application of the TLM the LV becomes too large it may be scaled down to avoid computational blow up. Additional LVs can be obtained by the same procedure, except that after each time step the perturbation has to be orthogonalized with respect to the subspace of the previous LVs, since otherwise all the LVs would converge to the leading LV.