

An Ensemble Data Assimilation System for POP

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(Thanks to Jeff Anderson, Nancy Collins, & Tim Hoar of NCAR)

Outline

Motivation

Assimilation Algorithm

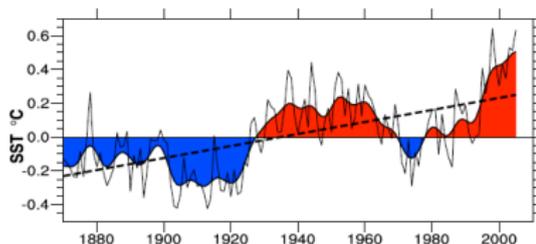
The 1990-91 DA Experiment

Some Improvements

Conclusions

Motivation

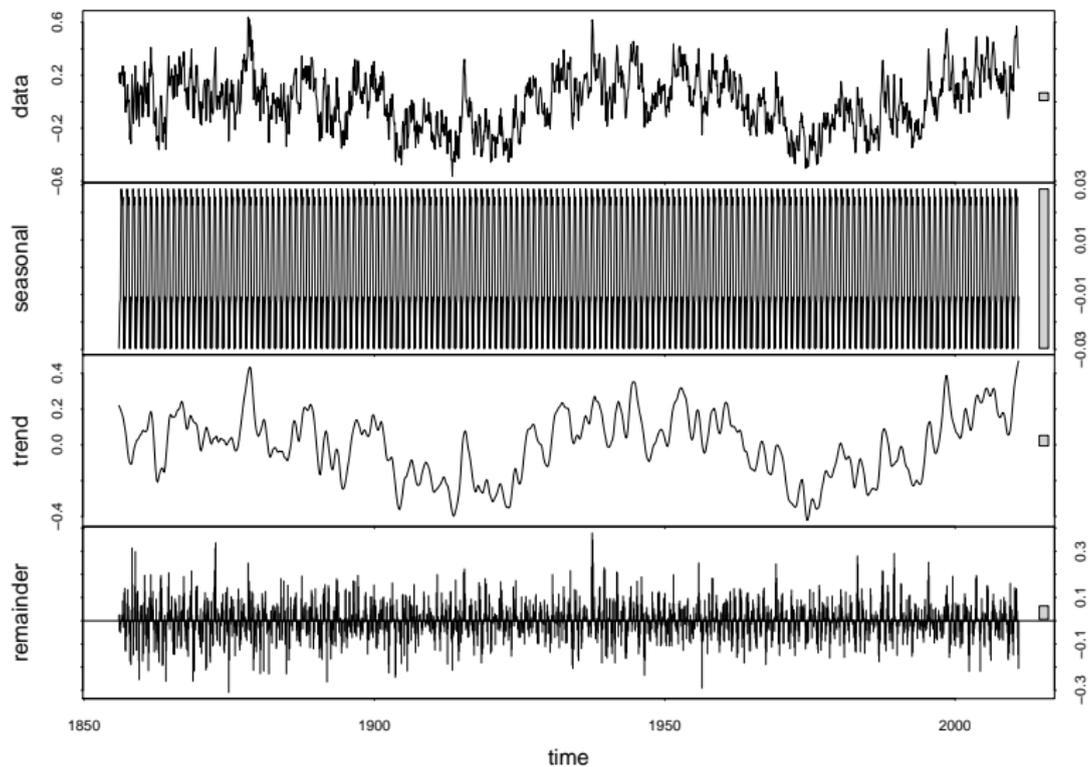
- ▶ Given its dynamical inertia, certain slow modes of global ocean circulation (e.g., AMO) are expected to be predictable on the interannual to decadal timescale



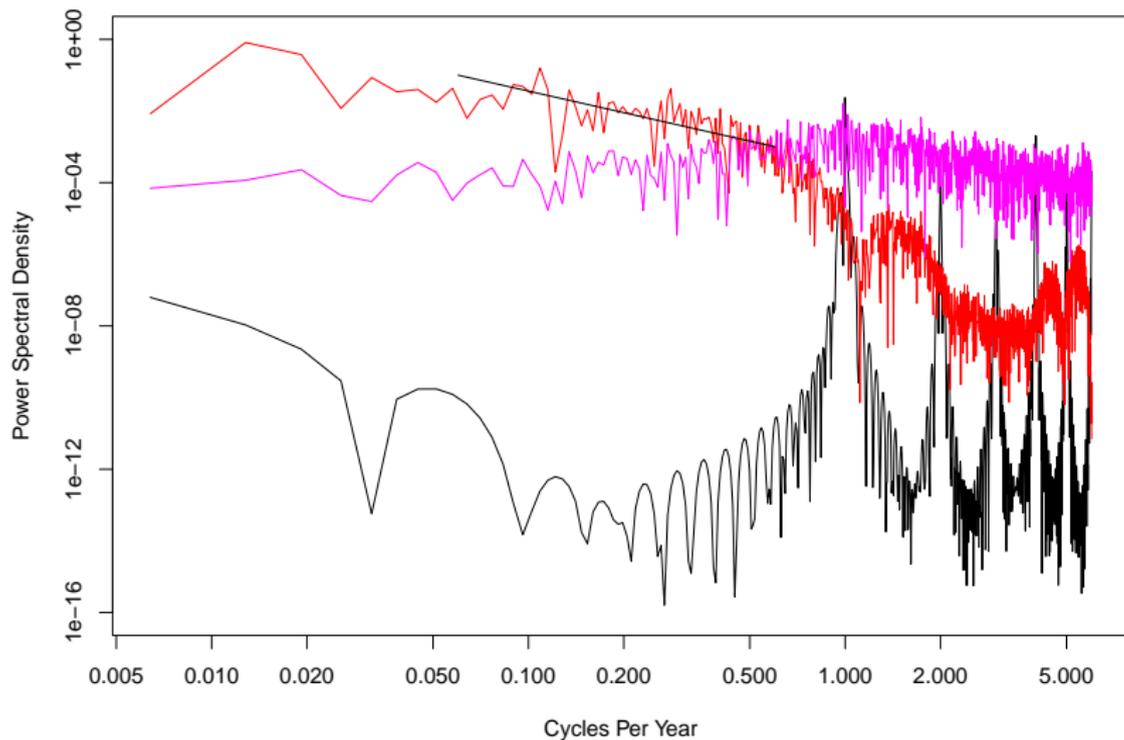
From Trenberth & Shea, 2006:
Annual SST anomalies averaged over the North Atlantic (0 to 60 N, 0 to 80 W) for 1870–2005, relative to 1901–1970 (C)

- ▶ A pre-requisite to using the “extended predictability of slow modes” is a successful assimilation of data to estimate the state of the ocean including the phase and amplitude of the slow modes.
- ▶ Given recent improvements in methodology and other reasons, we have chosen to develop an ensemble DA system for POP

Atlantic Multi-decadal Oscillation

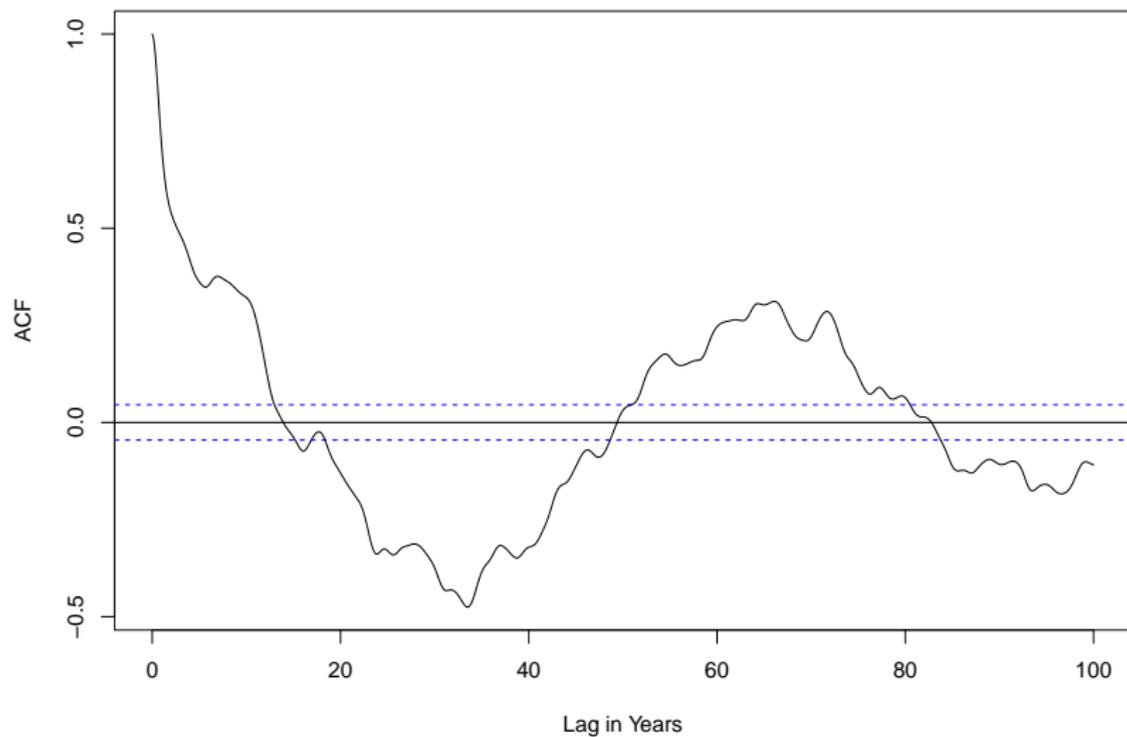


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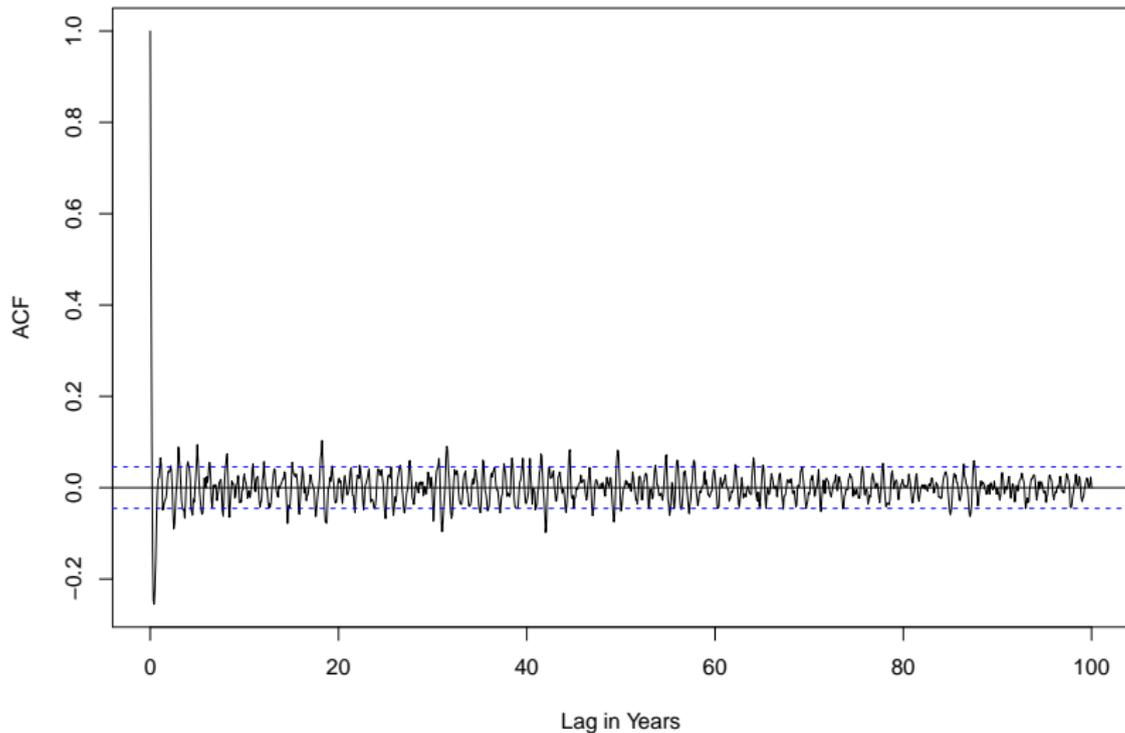
Atlantic Multi-decadal Oscillation

Series dcmp\$time.series[, 2]



Atlantic Multi-decadal Oscillation

Series `dcmp$time.series[, 3]`



Assimilation Algorithm I

For ob. y_{ob} with ob+representation error σ_{ob} ,
and given prior ensemble $\mathbf{y} = h(\mathbf{x}_m)$ (bold: ensemble vector)
 $m = 1, \dots, state_dim$. Use least squares:

- ▶ Compute posterior spread: $\frac{1}{\sigma_{po}^2} = \frac{1}{\sigma_{pr}^2} + \frac{1}{\sigma_{ob}^2}$
- ▶ Compute posterior ensemble mean: $\frac{\bar{y}_{po}}{\sigma_{po}^2} = \frac{\bar{y}_{pr}}{\sigma_{pr}^2} + \frac{y_{ob}}{\sigma_{ob}^2}$
- ▶ Compute ob. incr. for ensemble members (shift & compact):
$$\Delta \mathbf{y} = \bar{y}_{po} - \bar{y}_{pr} + \left(\frac{\sigma_{po}}{\sigma_{pr}} - 1 \right) \Delta \mathbf{y}_{pr}$$
- ▶ Regress ob. incr. onto state variable incr.
$$\Delta \mathbf{x}_m = \beta(\mathbf{y}, \mathbf{x}_m) \Delta \mathbf{y}$$

where $\beta(\mathbf{y}, \mathbf{x}_m) = cov(\mathbf{y}, \mathbf{x}_m) / \sigma_{pr}^2$
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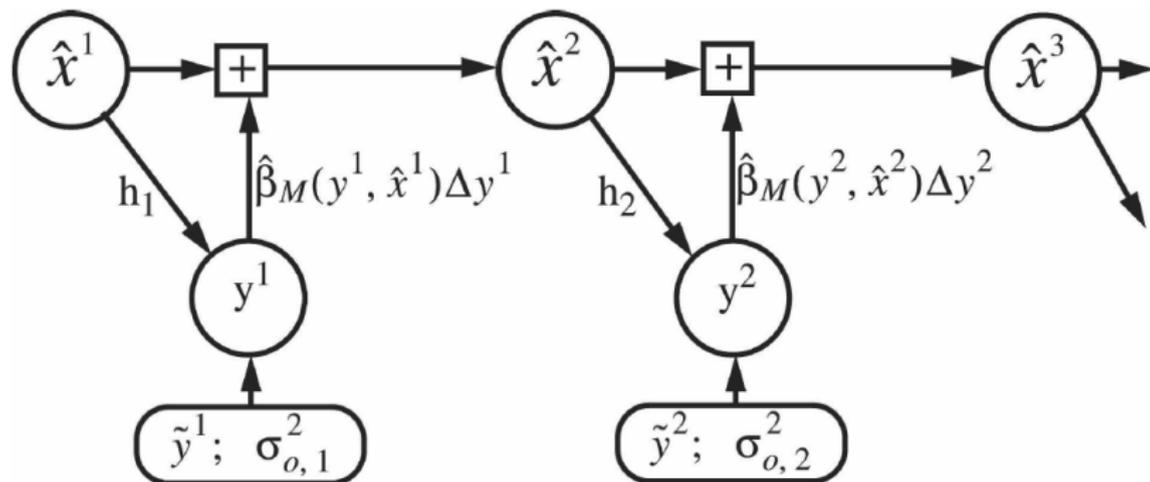


FIG. 1. A schematic depiction of the sequential filter algorithm. The forward observation operator for the first observation, h_1 , is applied to the ensemble state vector, \hat{x}^1 , to produce a prior ensemble approximation, y^1 , of the observation. Observation increments, Δy^1 , are computed using the observation value, y^1 , and error variance, $\sigma_{o,1}^2$, and regression is used to compute increments for the state, $\hat{\beta}_M(y^1, \hat{x}^1) \Delta y^1$. The state is updated by adding the increments to produce \hat{x}^2 and the process is repeated for each observation in turn.

(From Anderson & Collins, 2007)

Assimilation Algorithm III

Tippett et al., 2003 compare 3 deterministic SRFs

$$\text{ETKF: } \mathbf{Z}^a = \mathbf{Z}^f \mathbf{T} = \mathbf{Z}^f \mathbf{C} (\Gamma + 1)^{-1/2} \quad (16)$$

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From the point of view of flow instabilities,
there are important differences:

- ▶ Analysis perturbations in ETKF are linear combinations of (ens. no. of) forecast perturbations (each state vector component is similarly reconstituted/recombined.)
- ▶ Different state vector components of ensemble perturbation scale differently with EAKF (the adjustment matrix is of size state_dim x state_dim.)

$$\text{▶ } \Delta \mathbf{x}_{m,po} = \Delta \mathbf{x}_{m,pr} + \beta(\mathbf{y}, \mathbf{x}_m) \left(\frac{\sigma_{po}}{\sigma_{pr}} - 1 \right) h(\Delta \mathbf{x}_{m,pr})$$

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Spatio-Temporally Adaptive Inflation in DART

- ▶ Use y_{ob} , \mathbf{y} , and σ_{ob} via Bayes theorem to improve $\lambda(x_m, t)$ [Anderson, 2009]
- ▶ Prior PDF for $\lambda(x_m)$ is multivariate normal. Sequentially update each inflation factor
- ▶ Assume $cov(\lambda(x_{m_1}), \lambda(x_{m_2})) = cov(x_{m_1}, x_{m_2})$
- ▶ No time evolution of inflation factor (Persistence)
- ▶ Assuming no bias, obtain posterior mean of λ using approximations in Bayes formula; hold σ_λ fixed
 $N(\bar{\lambda}_{po}, \sigma_\lambda^2) = \frac{1}{\sqrt{2\pi}\theta} \exp(-D^2/2\theta^2) N(\bar{\lambda}_{pr}, \sigma_\lambda^2)$ where
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Setup of the DA Experiment

- ▶ LANL POP with Data Assimilation Research Testbed [Anderson et al., 2009]
- ▶ Corrected CORE (Coordinated Ocean-ice Reference Experiment) version 2 Interannual Forcing [Large and Yeager, 2009, Griffies et al., 2009]
- ▶ Weak salinity restoring; strong SST restoring under ice
- ▶ 20 member ensemble with spatiotemporally adaptive inflation [Anderson, 2009]; localization radius of 1100 km
- ▶ ICs from Jan 1 of different years of a control run with Normal Year Forcing
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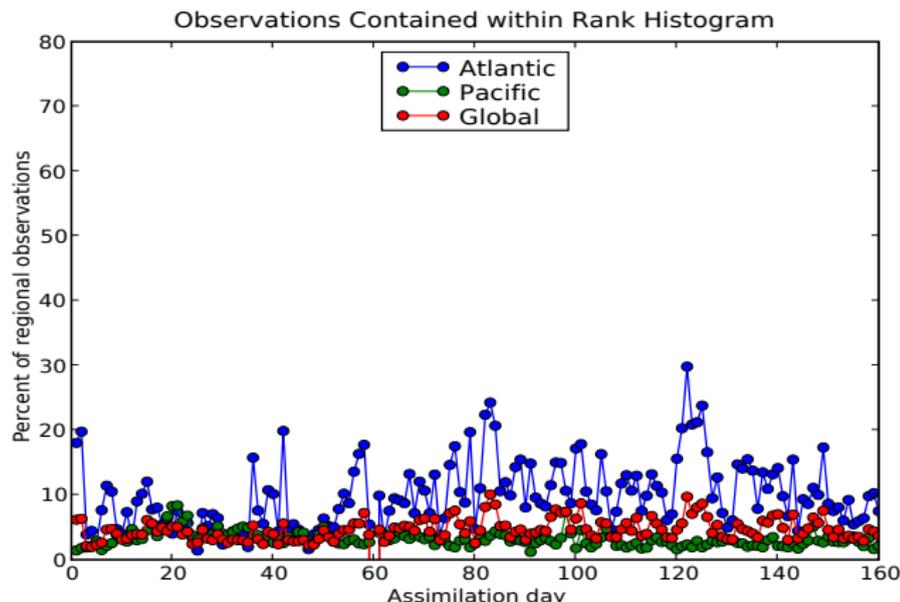
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DA system performs poorly with too few obs. ($< 10\%$) being contained in the ensemble

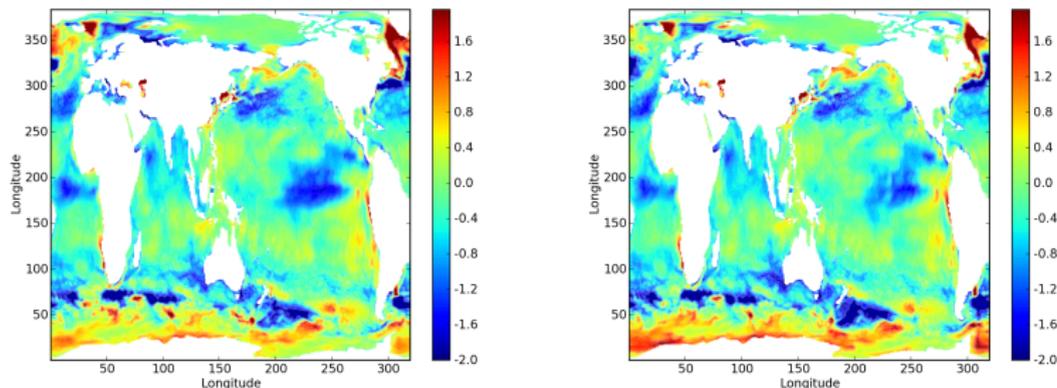
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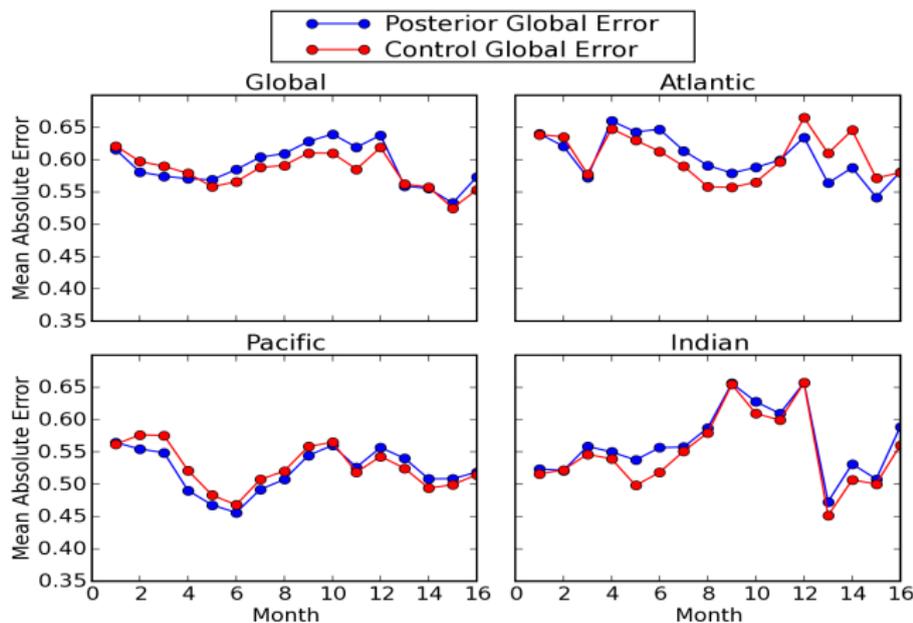
Insufficient ensemble spread in spite of sophisticated, spatio-temporally adaptive, statistical inflation [Anderson, 2009]

Analysis of 1990-91 DA Experiment II



Time-avgd difference wrt NOAA OI SST v2 **Left:** Control Run; **Right:** Assimilation Run. Cold bias in tropics and midlatitudes and warm bias at high latitudes and upwelling regions. **No Significant Improvement with DA.**

Analysis of 1990-91 DA Experiment III



Area-weighted Mean Absolute Error of the monthly-averaged SST anomaly for Jan 1990 through April 1991. In effect, no net reduction in error is seen with respect to the control.

Improvements to the DA System

- ▶ Enhance spread (S) based on background variability (σ):
 $S \rightarrow (1 - c)S + c\sigma$
 - ▶ Analogous to hybrid methods to boost rank of the forecast error covariance matrix: Boost under-estimated, ensemble-based, flow-dependent covariance with an *a priori*, background estimate.
- ▶ Simple bias correction (not yet analyzed)
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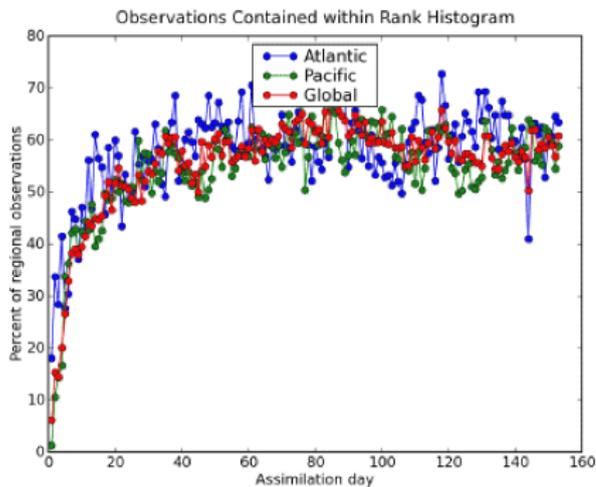
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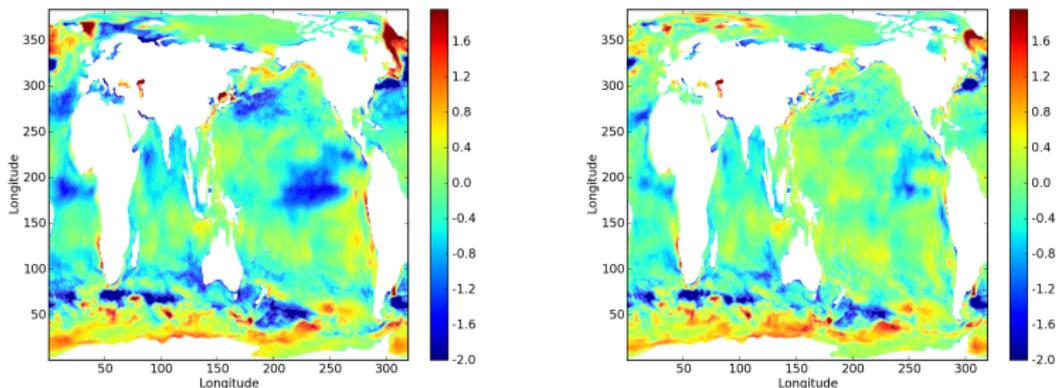
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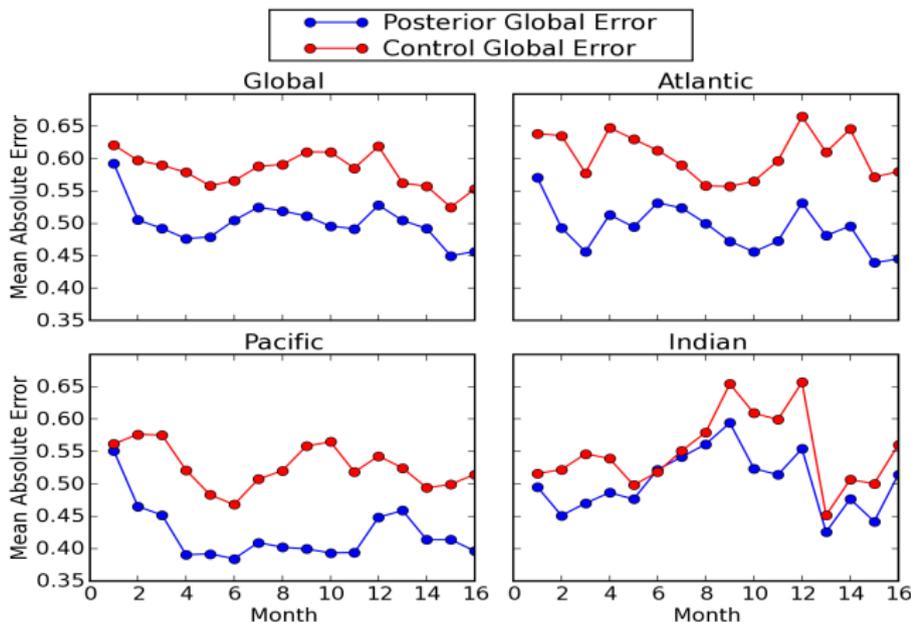
About 60% obs. used compared to < 10% previously

Improved Assimilation I

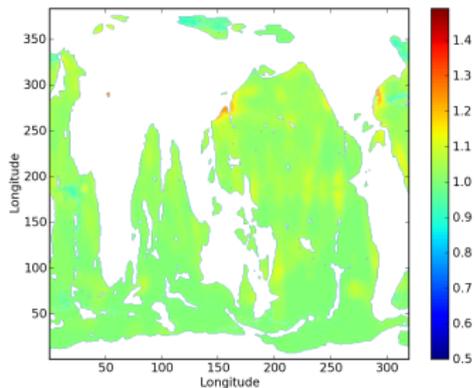
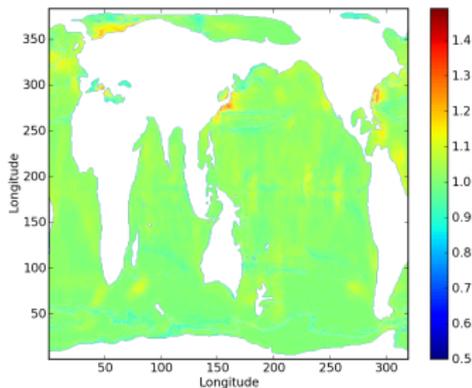
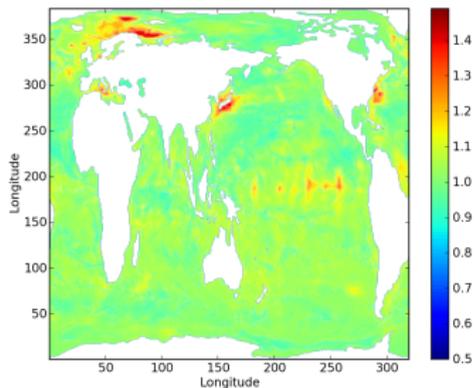
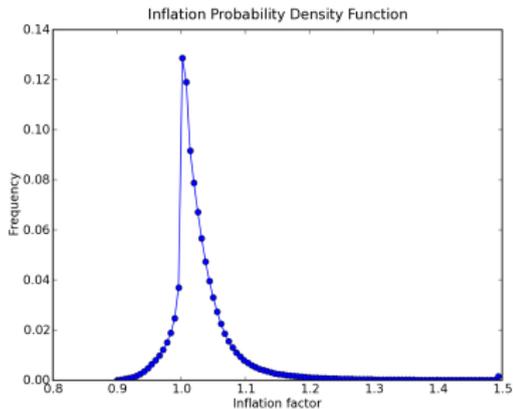


Time-avgd. difference wrt NOAA OI SST V2 **Left:** Same Control Ensemble Run; **Right:** Assimilation Run with Improvements.
Significant Improvement over Control Ensemble.

Improved Assimilation II

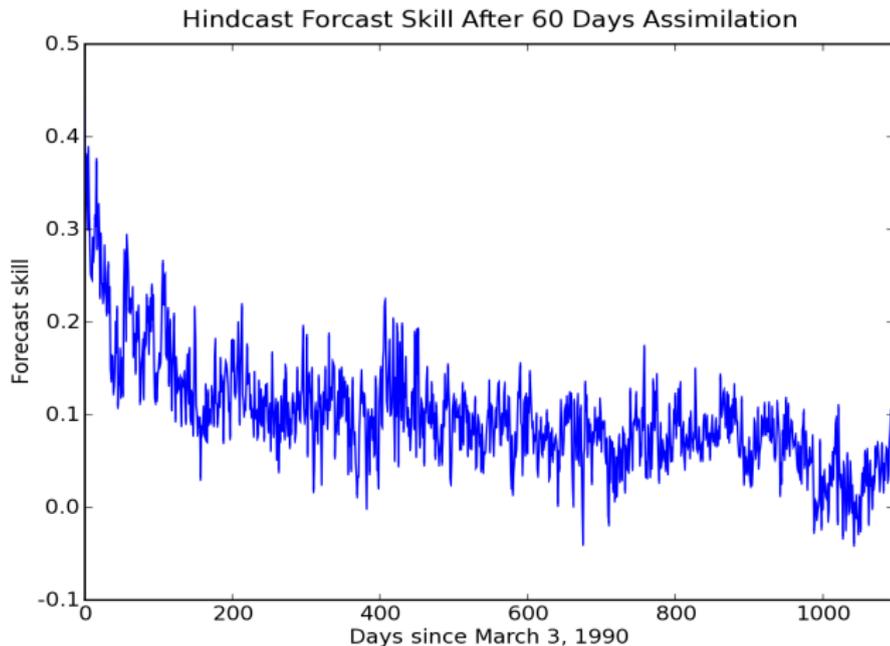


Area-weighted Mean Absolute Error of the monthly-averaged SST anomaly for Jan 1990 through April 1991. **Significant reduction in error with respect to control ensemble.**



Avg. inflation factors for pot. temp. at surface (top right) and depths of 1000 m (bottom left) and 3200 m

Hindcast



Faster loss in skill over first six months, followed by a slower decay

Conclusions

- ▶ Modifying spread to include a small fraction of background variability can be useful to improving ensemble diversity
 - ▶ Analogous to hybrid methods to boost rank of the forecast error covariance matrix.
- ▶ While the most serious disadvantage may seem to be a deleterious effect on the spread-skill relation, hindcasts using the ensemble-mean assimilated state shows significant skill

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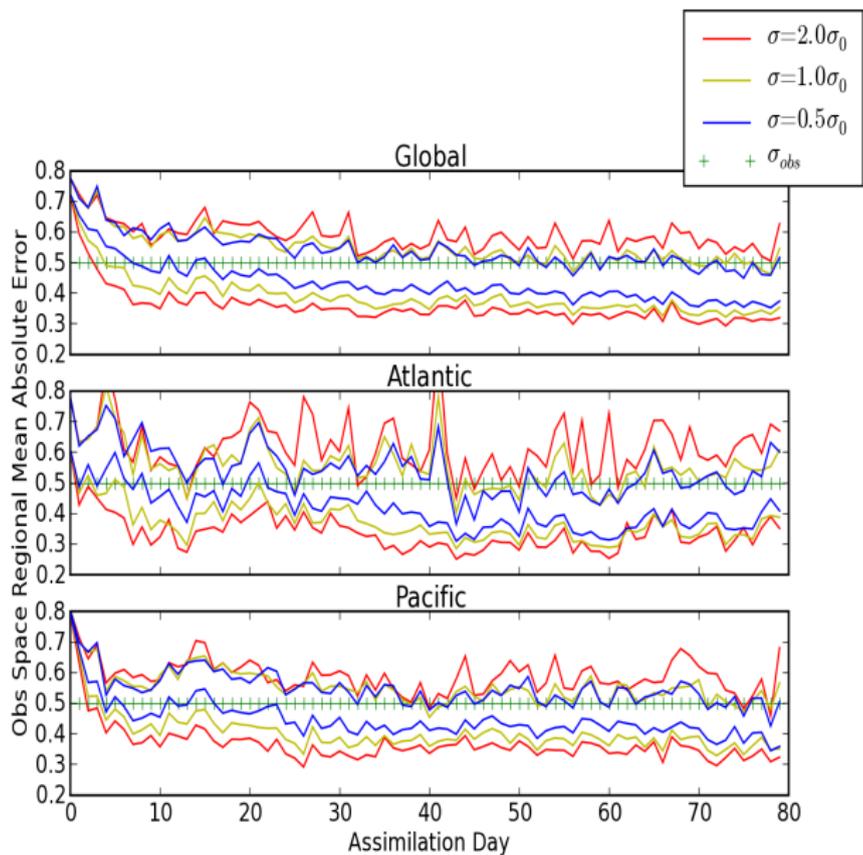
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Dependence on Amplitude of Specified Background Variability



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