OSSE-OSE activities with Multivariate Ocean Variational Estimation (MOVE) System.

I: Application of singular vector analysis to the Kuroshio large meander.

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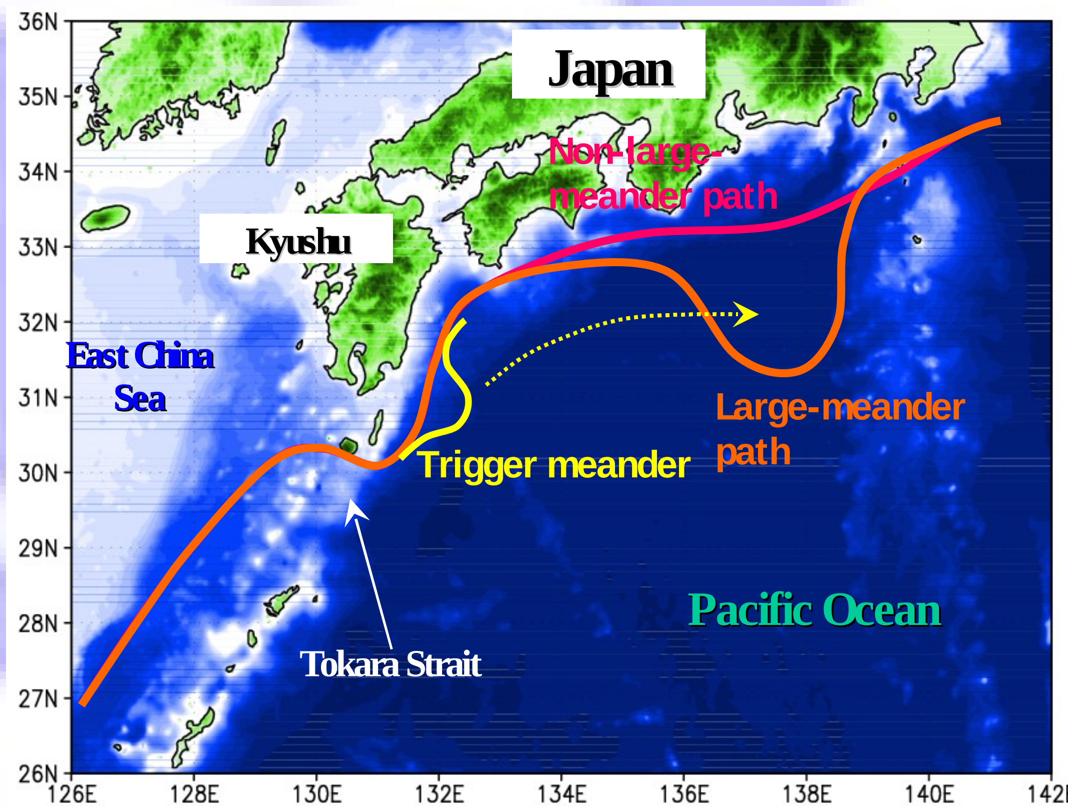
1. Introduction

☆ Introduction

Purpose

To investigate what triggers the formation of the Kuroshio large meander path using singular vector (SV) analysis (To identify the trigger objectively).

Information for constructing an effective observing system



Outline

1. Introduction
2. Strategy
3. The 1st SV
4. Summary

2. Strategy

★ Basic property of Singular Vector (SV)

State vector in the final state (t=n)

$$\mathbf{x}_n = M(\mathbf{x}_0)$$

Linear deviation in the final state

$$\Delta\mathbf{x}_n = \mathbf{L}\Delta\mathbf{x}_0$$

$\mathbf{L}$: Tangent Linear model

The Right Singular Vector (RSV) of $\mathbf{L}$ is the eigen vector of $\mathbf{L}^T\mathbf{L}$.

$$\mathbf{L}^T\mathbf{L}\mathbf{u}_i = \lambda_i^2\mathbf{u}_i$$

$\mathbf{u}_i$: RSV λ_i: Singular Value

RSV evolves to the Left Singular Vector (LSV) by the tangent linear model, and LSV evolves back to the RSV by the adjoint model.

$$\mathbf{L}\mathbf{u}_i = \lambda_i\mathbf{v}_i, \quad \mathbf{L}^T\mathbf{v}_i = \lambda_i\mathbf{u}_i$$

$\mathbf{v}_i$: LSV $\mathbf{L}^T$: Adjoint model

The 1st RSV, $\mathbf{u}_1$, associated with the largest singular value, λ_1, maximizes the growth ratio of the perturbation, r.

$$r^2 = \frac{\Delta\mathbf{x}_n^T\Delta\mathbf{x}_n}{\Delta\mathbf{x}_0^T\Delta\mathbf{x}_0} = \frac{\Delta\mathbf{x}_0^T\mathbf{L}^T\mathbf{L}\Delta\mathbf{x}_0}{\Delta\mathbf{x}_0^T\Delta\mathbf{x}_0}$$

$$\max(r^2) = \mathbf{u}_1^T\mathbf{L}^T\mathbf{L}\mathbf{u}_1 = \lambda_1^2$$

☆ Singular Vector for sensitivity analysis

Deviation restricted in the target (Kuroshio meandering) region

$$\Delta\mathbf{z}_n = \mathbf{P}\Delta\mathbf{x}_n = \mathbf{PL}\,\Delta\mathbf{x}_0 = \mathbf{PLT}\,\Delta\mathbf{y}_0$$

P : Projection Matrix $\mathbf{y}_0$: parameters for initial state

T: Transformation Matrix from $\Delta\mathbf{y}_0$ to $\Delta\mathbf{x}_0$

Ratio of the magnitude of target, $\Delta\mathbf{z}_n{}^{T}\Delta\mathbf{z}_n$ (Final Norm) to the magnitude of the deviation of the initial parameters, $\Delta\mathbf{y}_0{}^{T}\Delta\mathbf{y}_0$ (Initial Norm).

$$r^2 = \frac{\Delta\mathbf{z}_n^T\Delta\mathbf{z}_n}{\Delta\mathbf{y}_0^T\Delta\mathbf{y}_0} = \frac{\Delta\mathbf{y}_0^T(\mathbf{PLT})^T(\mathbf{PLT})\Delta\mathbf{y}_0}{\Delta\mathbf{y}_0^T\Delta\mathbf{y}_0}$$

The 1st RSV of **PLT**, $\mathbf{u}_1$, maximizes the ratio r.

$$\max(r^2) = \mathbf{u}_1^T(\mathbf{PLT})^T(\mathbf{PLT})\mathbf{u}_1 = \lambda_1^2$$

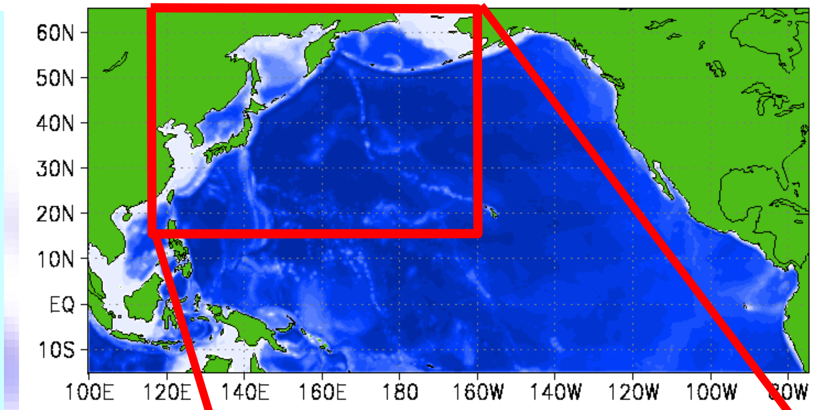
The state in the target region is most sensitive to the perturbation parallel to the 1st RSV in the initial-state parameter ($\Delta\mathbf{y}$) space.

★ Ocean Model (MRI.COM)

<u>MRI.COM: OGCM developed in MRI.</u>

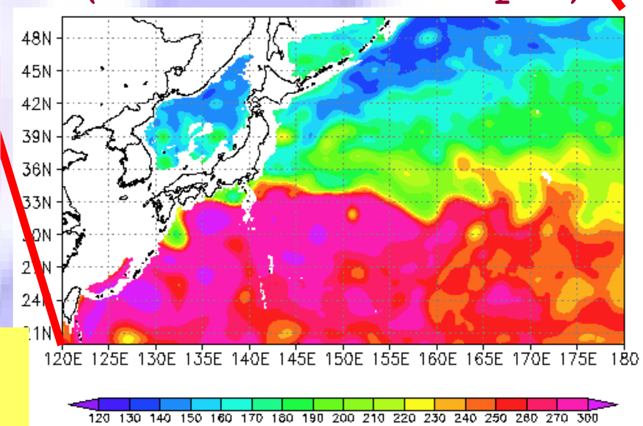
Grid: Arakawa B-grid
σ-z hybrid (σ for near surface)
Tracer: QUICK
harmonic diffusion
Momentum: Genralized Arakawa,
(conserving enstrophy)
Takano-Ohnishi Scheme
(diagonal advection
near sloping bottoms)
biharmonic Smagorinsky
+ harmonic viscosity
Forcing: NCEP-R2
Sea Ice → removed in the experiment

North Pacific Model (0.5 × 0.5



Western North Pacific Model
(0.1 × 0.1 near Japan)

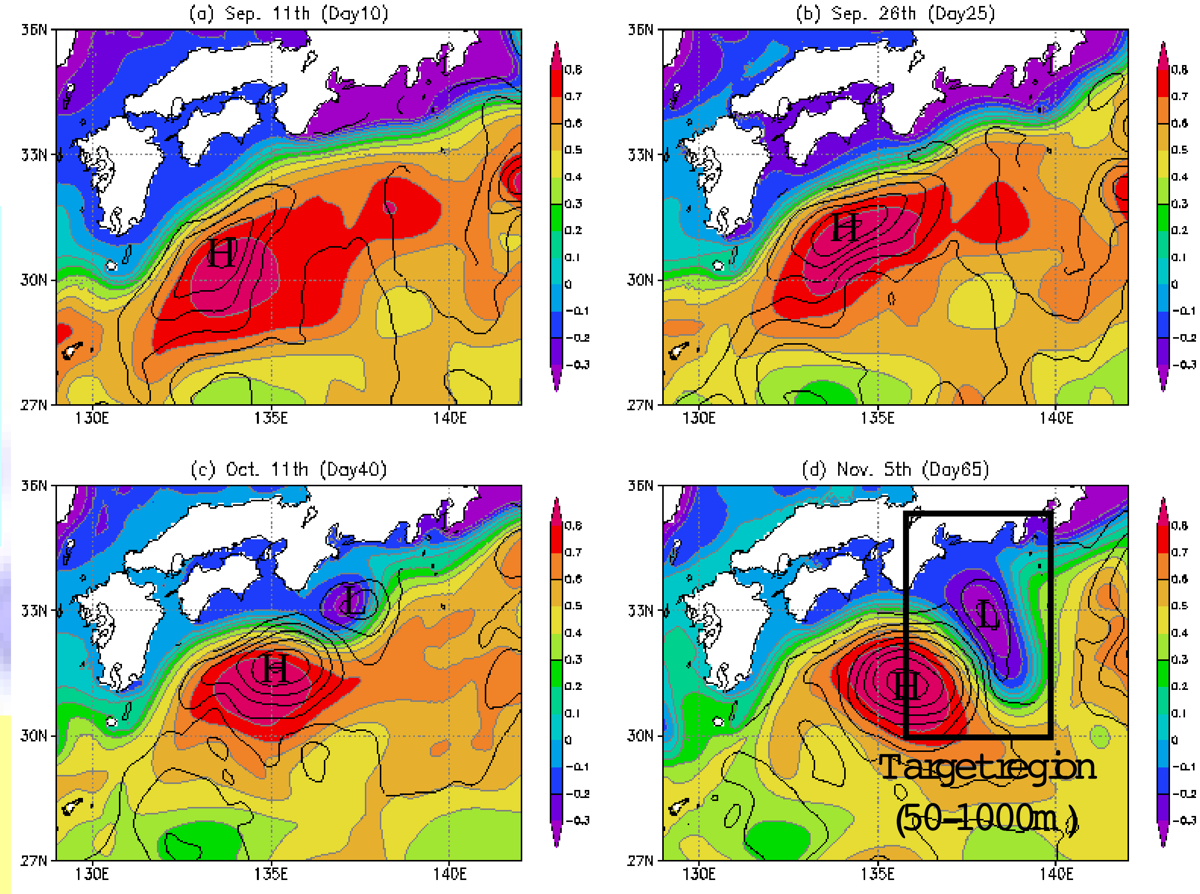
One-way nesting



Singular vector is calculated using the tangent linear and adjoint models of the western North Pacific model.

☆ Target

Simulated meander
(1996/09/01
- 1996/11/10)
Color: SSH
Contour: Pressure at 1800m



$\Delta z_n^T \Delta z_n$ (Final Norm)

→ Kinetic energy
in the target region

$$\Delta \mathbf{z}_n^T \Delta \mathbf{z}_n = \sum_i \sum_k V_{ik} (\Delta u_{ik}^2 + \Delta v_{ik}^2)$$

Δx_n

Δu, Δv
ΔT, ΔS, $\Delta \eta$

P

Δz_n

$V^{1/2}\Delta u$, $V^{1/2}\Delta v$
(in the target region)

0

★ Parameters for the initial state

Initial Time → about 2 month before the meandering occurs.

$\Delta \mathbf{y}_0^T \Delta \mathbf{y}_0$ (Initial Norm) → Kinetic + potential energy in the whole model region

$$\Delta \mathbf{y}_0^T \Delta \mathbf{y}_0 = \sum_i \left[\sum_k V_{ik} (\Delta u_{ik}^2 + \Delta v_{ik}^2 + N^2 \Delta h_{ik}^2) + g A_i \Delta \eta_i^2 \right]$$

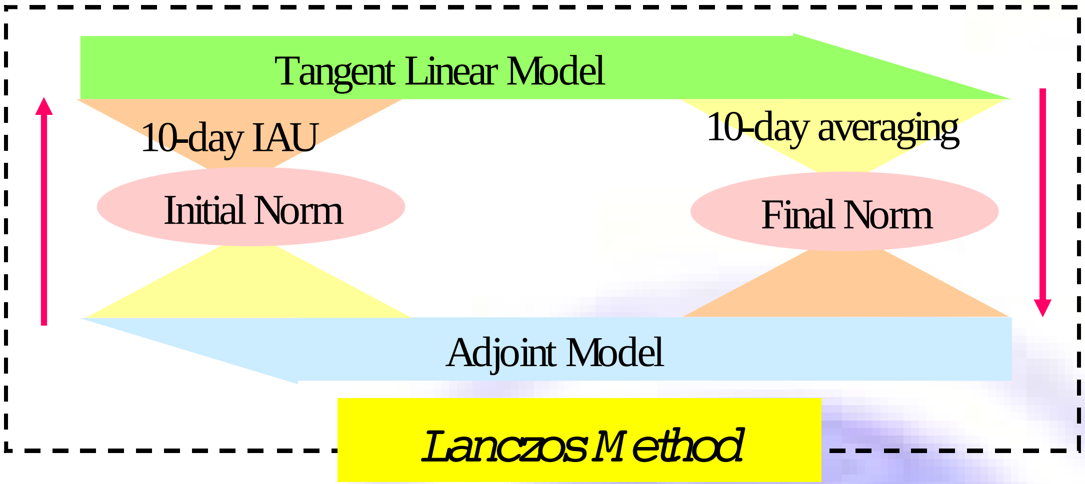
Δh_{ik}: vertical deviation of the water mass position

N : Brunt-Vaisala frequency

T-S relation in the vertical profile in the background is conserved.

$\Delta \mathbf{y}_0$	$\Delta \mathbf{x}_0$
$\Delta u', \Delta v'$	$V^{-1/2}\Delta u', V^{-1/2}\Delta v'$
$\Delta h'$	$N^{-1}V^{-1/2} h (\partial T/\partial Z) \Delta T$
	$N^{-1}V^{-1/2} h (\partial S/\partial Z) \Delta S$
$\Delta \eta'$	$g^{1/2} A^{-1/2} \Delta \eta'$

☆ Method of calculation



10-day IAU
→ Setup of initial perturbation
10-day averaging
→ For evaluating final norm

Excluding high-frequency noise

1st mode (singular value: $\sqrt{642}$), 2nd mode (singular value: $\sqrt{103}$)

Shown in the rest of the presentation

Simulation runs

TL run: RSV → Tangent Linear model

Adjoint run: LSV → Adjoint model

+SV run: Background + RSV → Original nonlinear model

–SV run: Background – RSV → Original nonlinear model

3. The 1st SV

★ Cyclic Property of SV

Arrow :400m Velocity
Color:1200m Temperature

RSV: Anticyclonic and warm anomaly south east of Kyushu.

↓

TL run: Cyclonic and cold anomaly is generated and propagated to the east.

↓

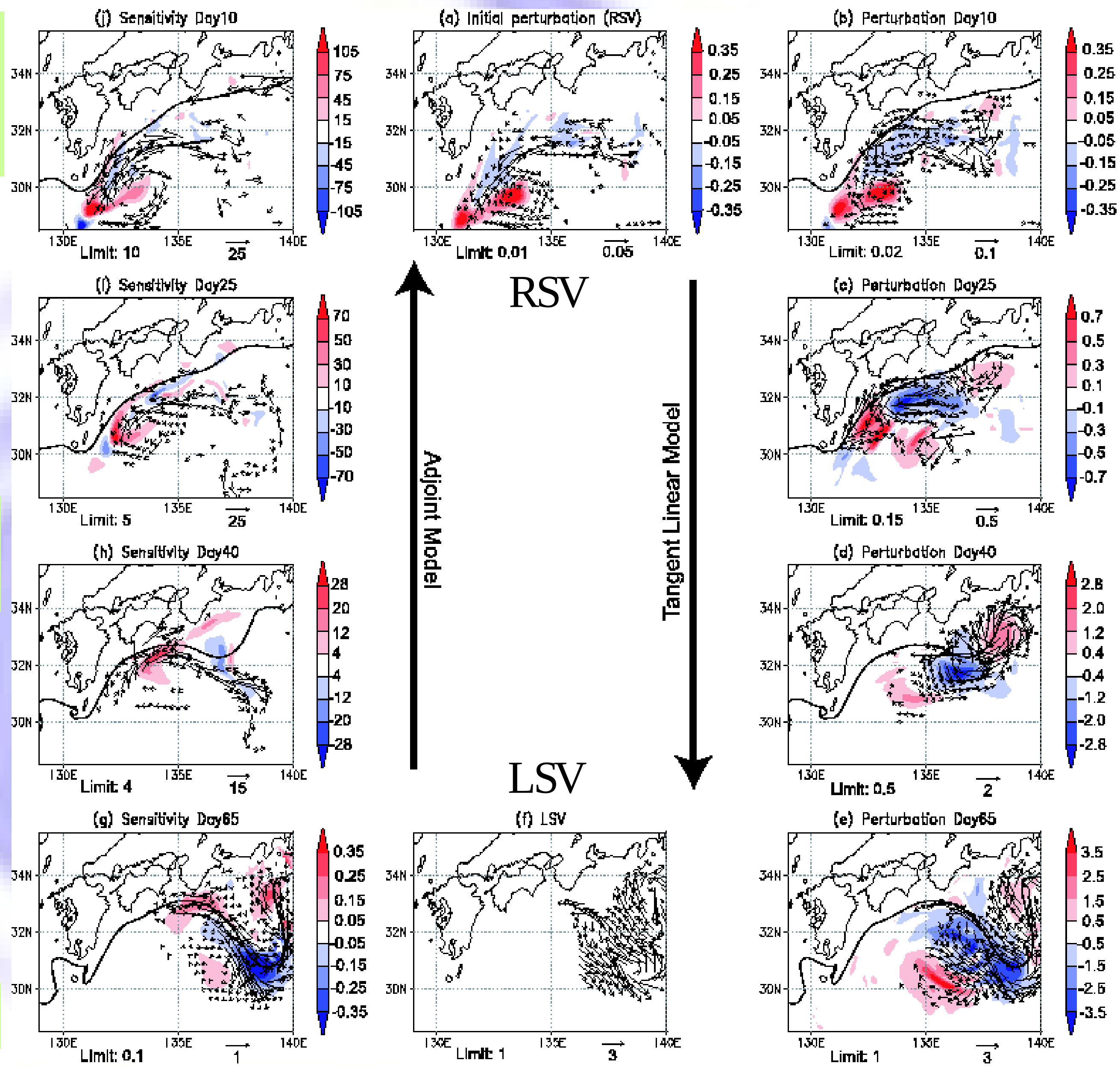
LSV: Cyclonic anomaly west of the meander.

↓

Adjoint run: Signal is propagated to the east. (The route is different from the TL model.)

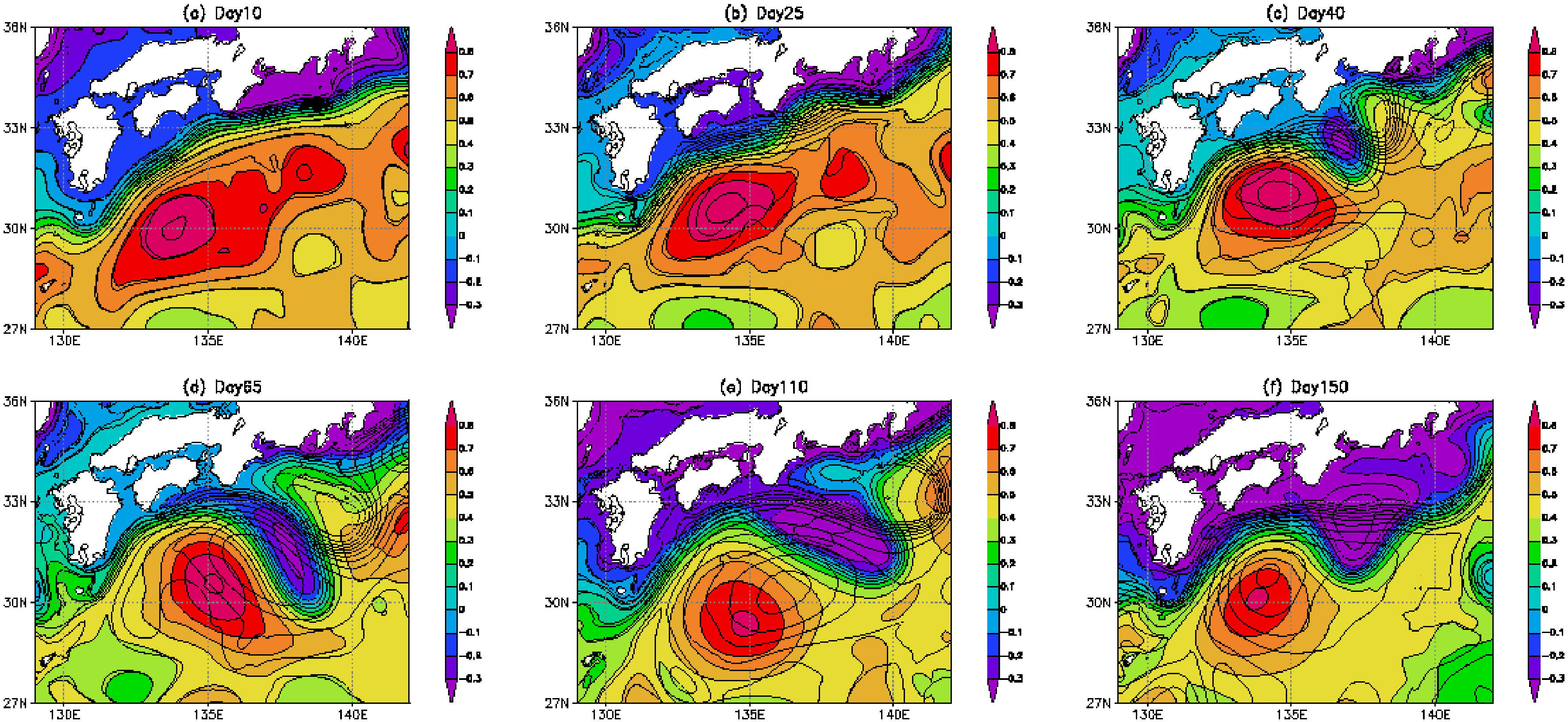
↓

RSV



★ Comparison of SSH in +SV and –SV runs

Color: +SV run, Contour: -SV run



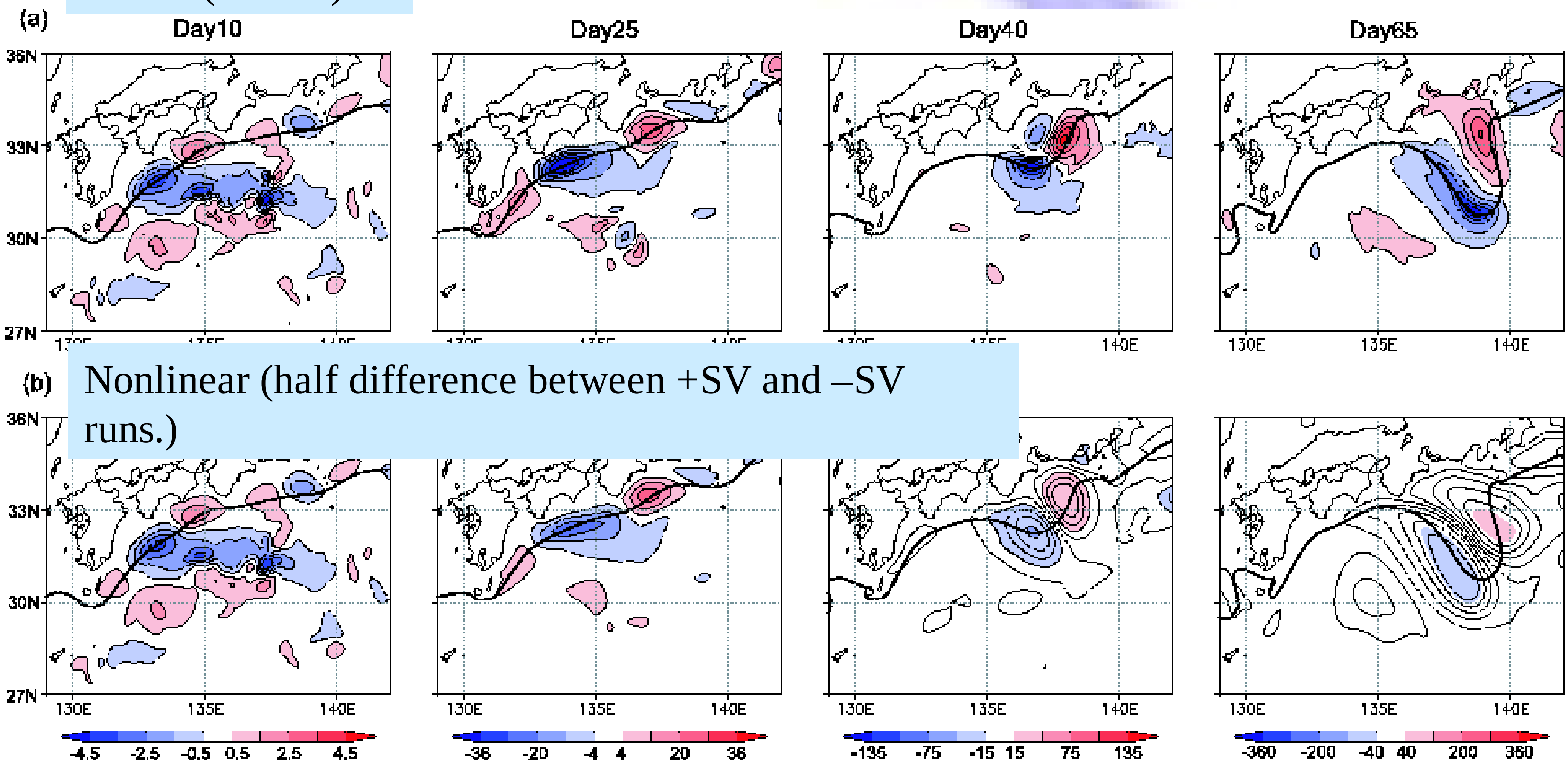
+SV run: the meander developed at Day65 and remained there.

-SV run: the meander propagates faster, and passes the 140E line at Day65,
The Kuroshio path gets straight at Day150.

★Linear and nonlinear evolution of SSH in SV

Linear (TL run)

Nonlinear (half difference between +SV and –SV runs.)

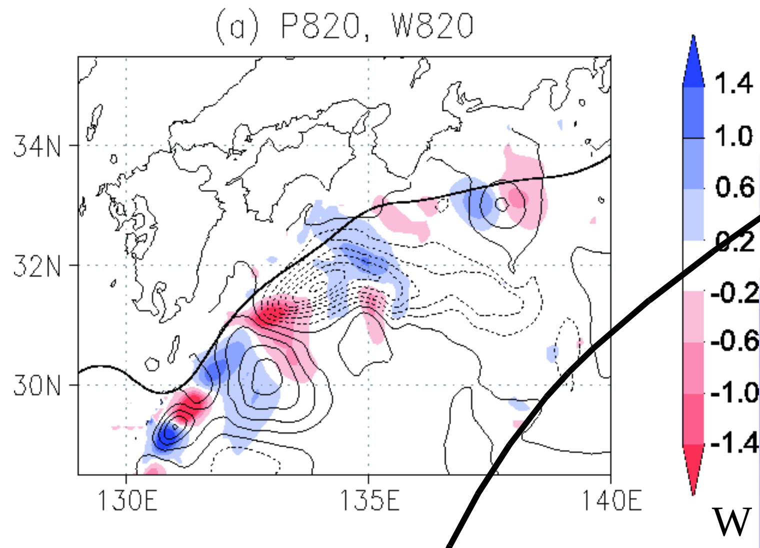


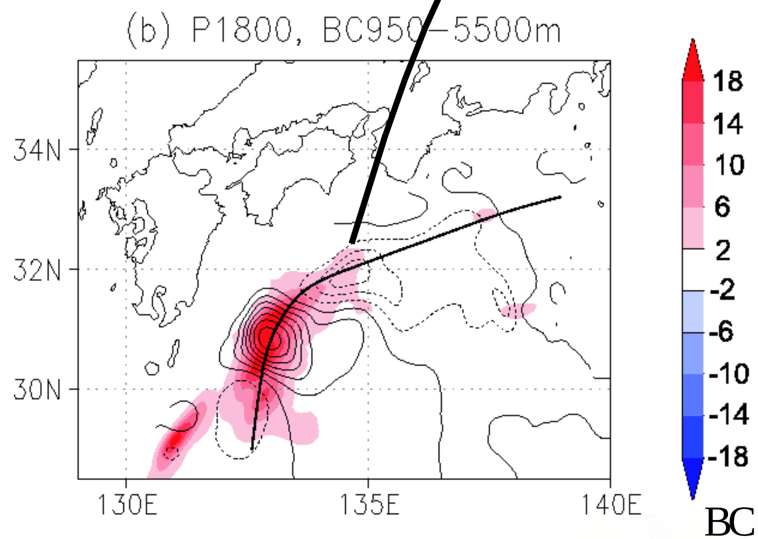
Linear and nonlinear evolutions are almost same until Day25.

Although the growth is suppressed in the nonlinear evolution after Day40, the position of anomaly is still similar.

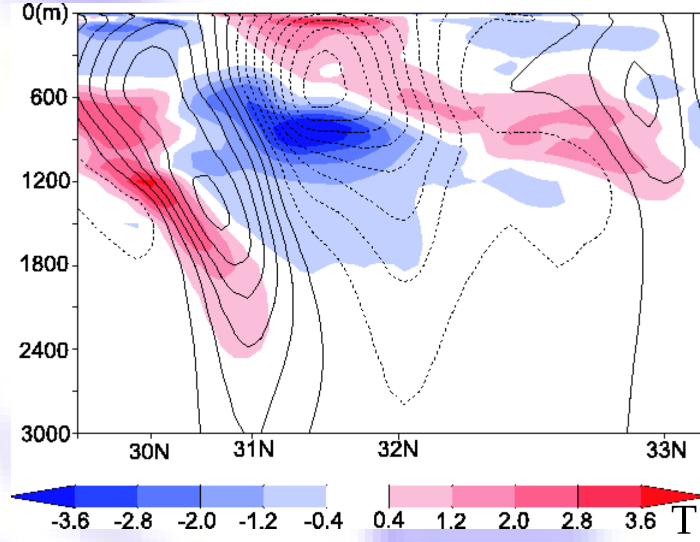
★Setup of the growth

State in Day10 in TL run





Vertical section of T and P



Upper layer

Cold advection across the Kuroshio current

Downwelling

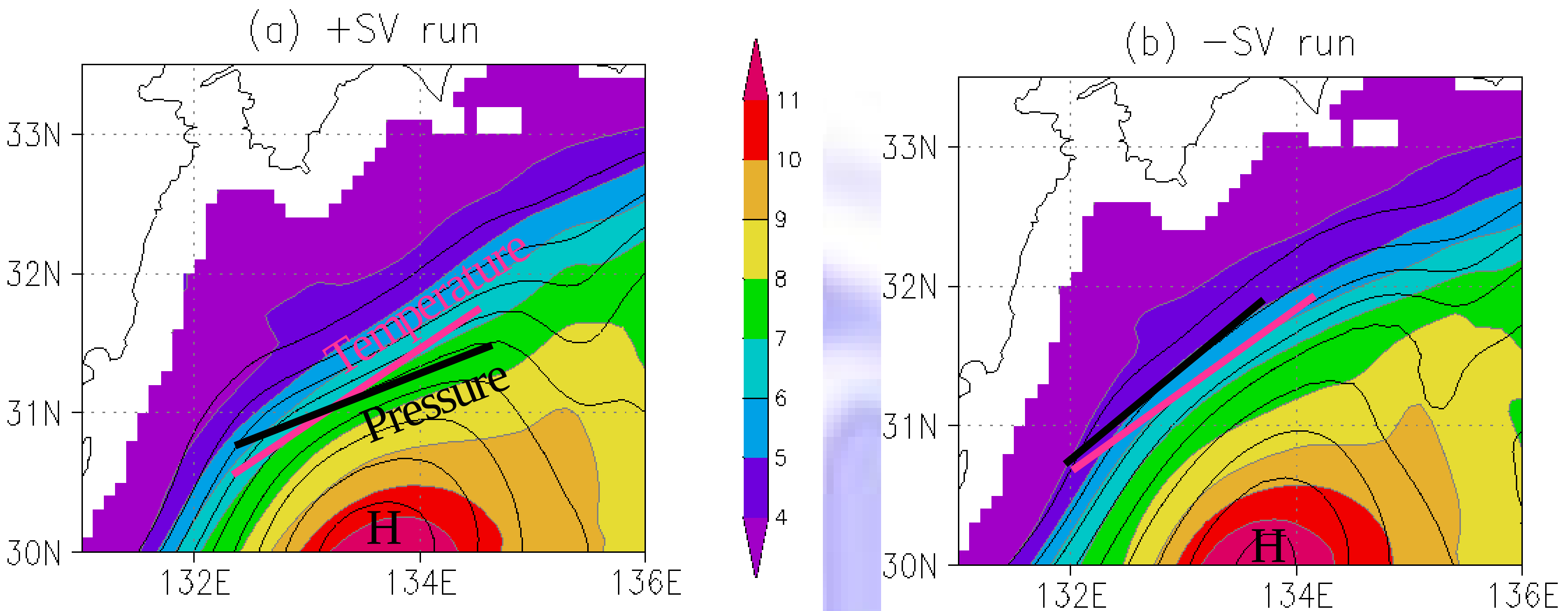
Lower layer

Generation of the anticyclonic eddy

BC: Baroclinic energy conversion rate (Background→Eddy) (Good indicator of baroclinic instability)

☆ Difference of setup between +SV and –SV runs

Temperature and pressure at 820m in Day10



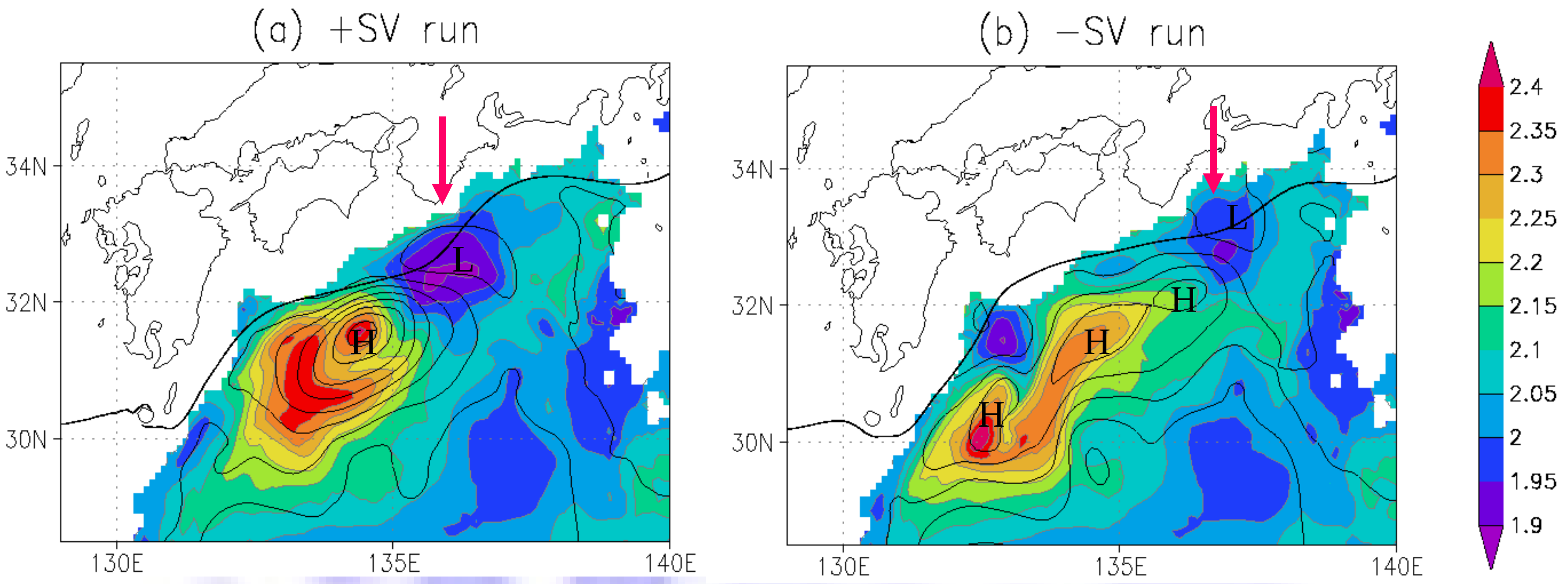
The pressure contours cross the isotherms in +SV run, which means cold advection across Kuroshio and downwelling exist there.

⟶ An anticyclonic eddy is enhanced in the lower layer.

The pressure contour is almost parallel to isotherms in –SV run.

☆ Difference in the deep layers

Temperature and pressure at 1800m in Day25



In +SV run, the deep anticyclonic eddy is developed and produces the cold anomaly which affects the pressure field in the upper layer.

The cold anomaly is not so definite in –SV run because the deep anticyclonic eddy does not developed.

★ Baroclinic energy conversion rate in +SV, –SV runs

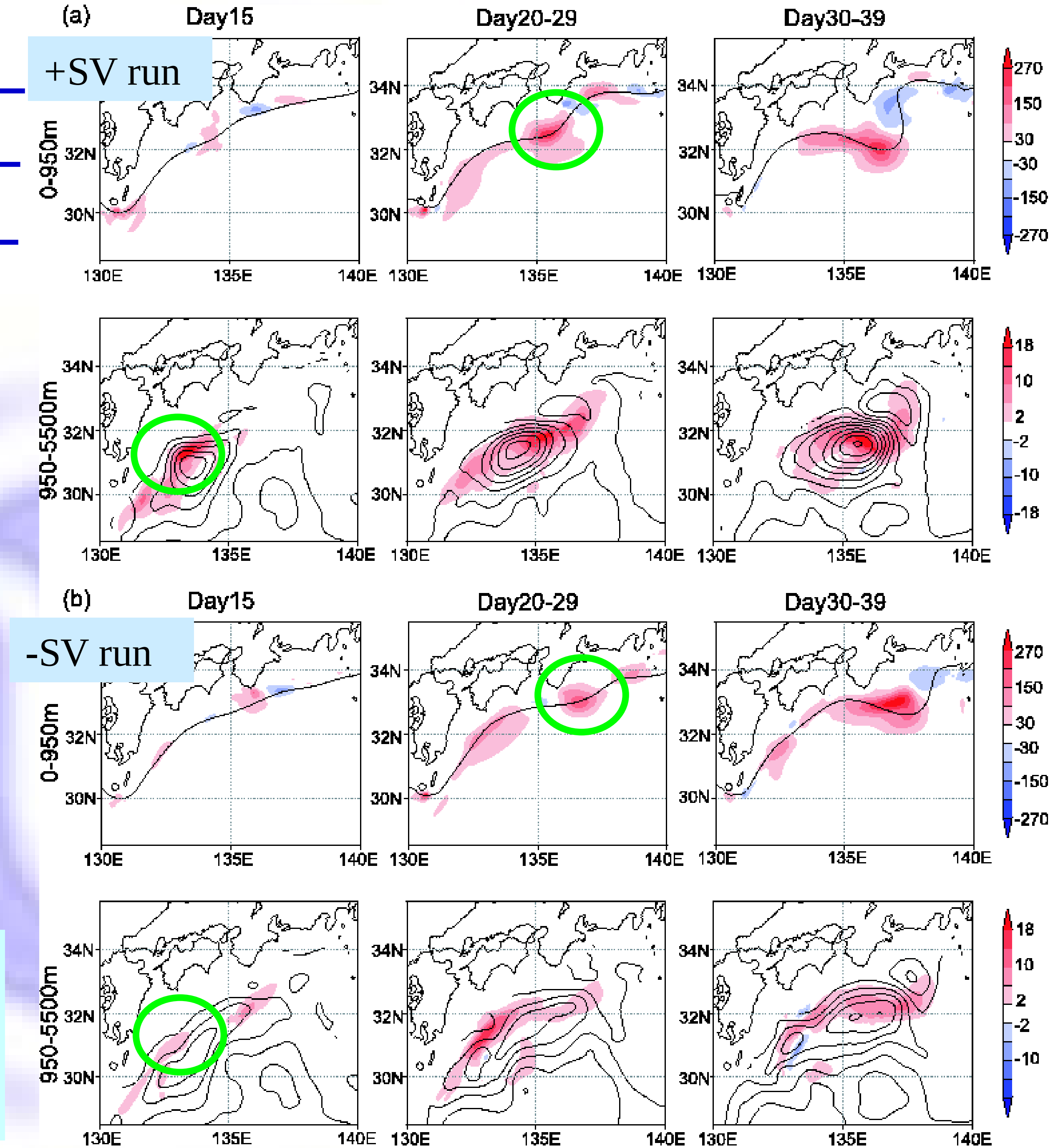
The large BC in the deep layer develops the anticyclonic eddy in +SV run.

Because of the large BC above the downstream edge of the deep anticyclonic eddy, the growth of meander starts early than in –SV run.

Color: BC

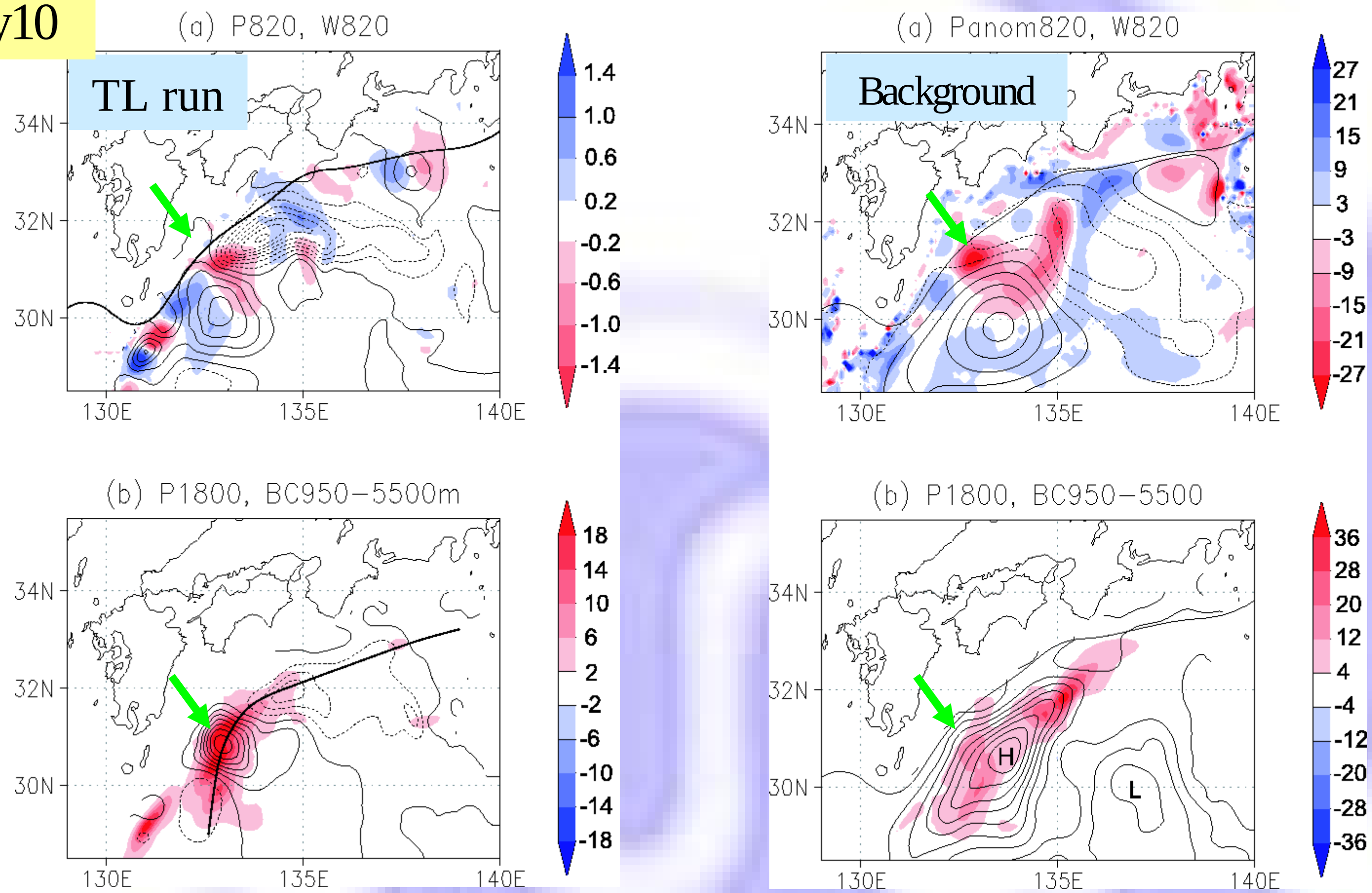
Thick line: Kuroshio Axis

Contour: 1800m pressure



★ Similarity of SV and background initial state

Day10



It may be because the initial of the background originally includes the seed (SV) that the large meandering occurs in the background.

★ "Contribution"

Adjoint variables

$$\tilde{\mathbf{x}}_i^T \Delta\mathbf{x}_i = (\mathbf{L}^T\tilde{\mathbf{x}}_{i+1})^T\Delta\mathbf{x}_i = \tilde{\mathbf{x}}_{i+1}^T\mathbf{L}\Delta\mathbf{x}_i = \tilde{\mathbf{x}}_{i+1}^T\Delta\mathbf{x}_{i+1}$$

$$\ldots = \tilde{\mathbf{x}}_n^T\Delta\mathbf{x}_n = (\mathbf{P}^T\Delta\mathbf{z}_n)^T\Delta\mathbf{x}_n = \Delta\mathbf{z}_n^T\mathbf{P}\Delta\mathbf{x}_n$$

$$= \Delta\mathbf{z}_n^T\Delta\mathbf{z}_n = (\text{Singular Value})^2$$

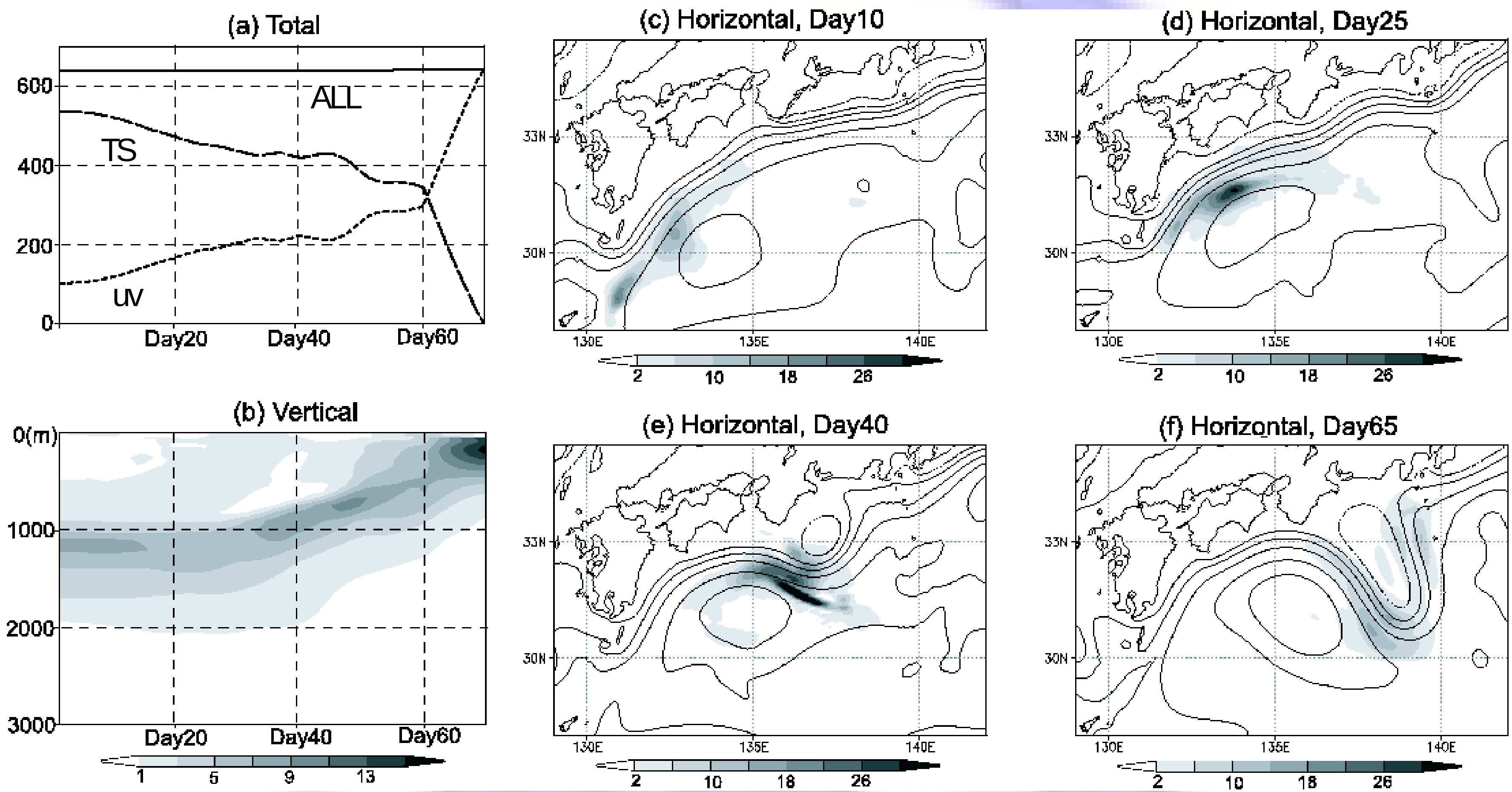
The sum of the product of forward variables (in TL run) and adjoint variables (in adjoint run) becomes the final norm (the square of the singular value), and conserved.

The product represents the ratio of the contribution of the elements (temperature, velocity, etc.) to the final norm, that is, the growth of the perturbation.

Therefore, we call this product "contribution".

Distribution of the "contribution" shows where or which element is important for the perturbation growth.

★ "Contribution" in the SV



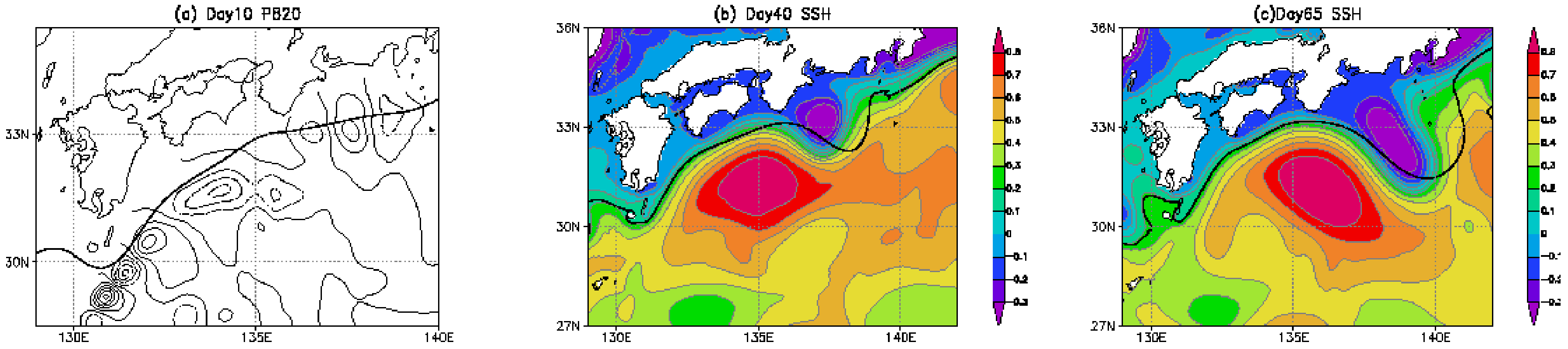
(Horizontally integrated)

(Vertically integrated)

Large Contribution → 1000-2000m and south east of Kyushu at Day10

☆ Removing high-frequency variability

Result with 1-day IAU for the initial norm and 1-day averaging for the final norm



(a): 820m pressure at Day10 in TL run, (b) SSH at Day40, (c) SSH at Day65 in –SV run. Thick black line: Kuroshio Axis in the –SV run of original (10-day IAU and averaging) result.

The initial state has smaller structure.

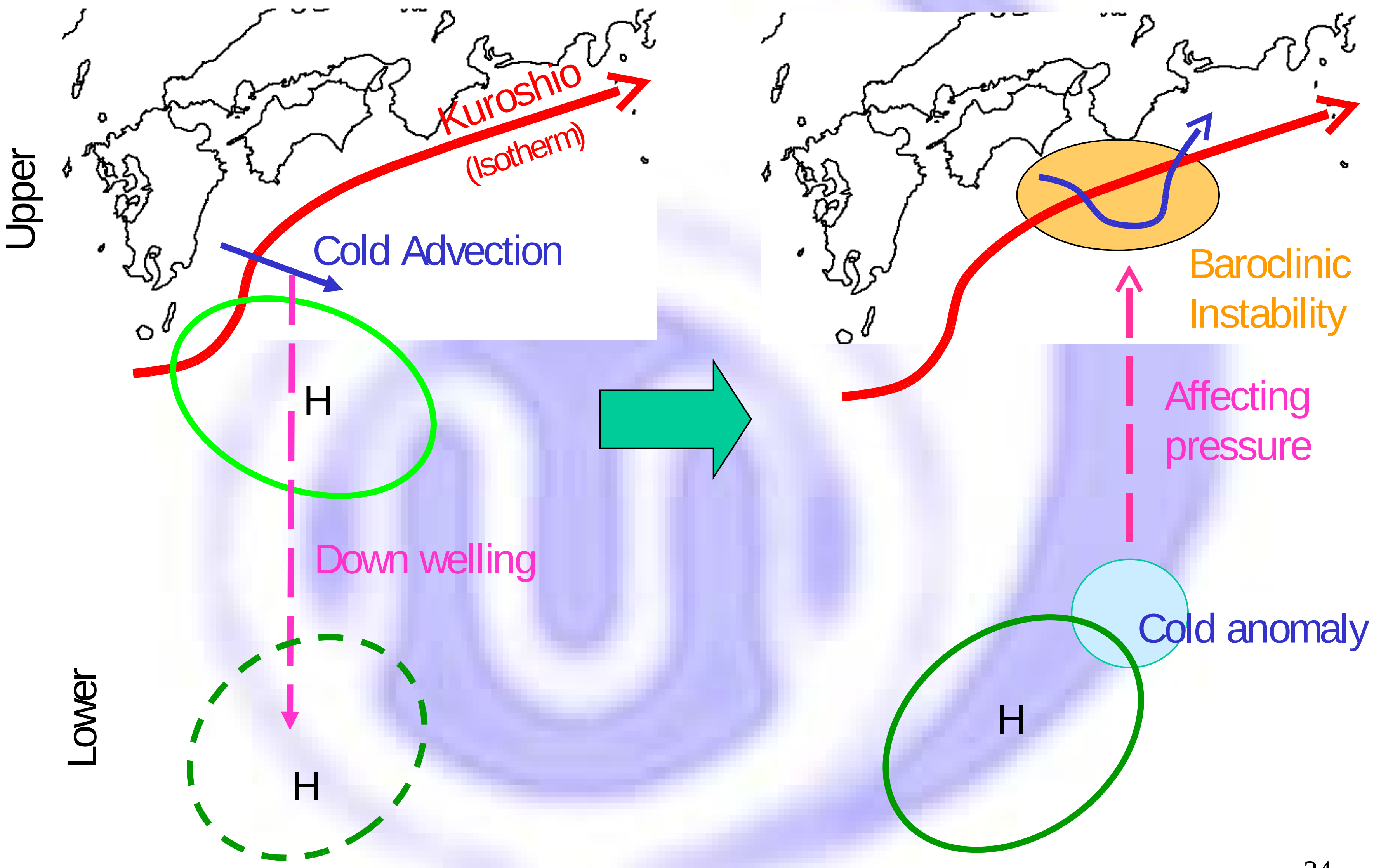
The small meander propagates not so fast as the original –SV run.

➔Difference between the background and –SV run is smaller than the original, although the singular value is larger than the original case.

The SV does not represent nonlinear physics properly.

4. Summary

★Mechanism of the meandering growth



☆ Summary

Where should we observe?

South east of Kyushu (Eddies interact with Kuroshio there.)

T and S in the layer between 1000-2000m depth

(It affects the pressure field in the upper layer.)

What is essential for performing the SV analysis?

How do we measure the magnitude of the initial and final perturbation?

(How do we chose the initial and final norms?)

Removing the high frequency variability.

(subtracting SV reflecting nonlinear physics.)

Can we neglect the influence of surface and boundary forcing?

(It is probably OK for the case of the Kuroshio meandering.)

Thank you!